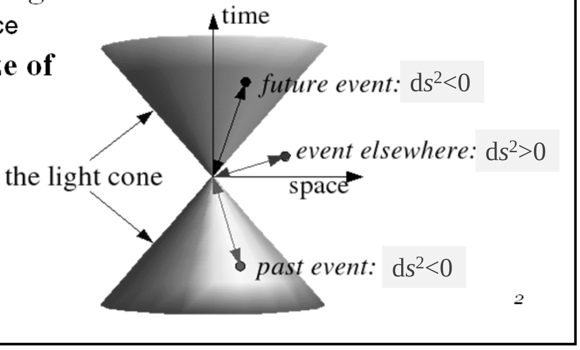
Modern cosmology 2: Type Ia supernovae and Λ

- **Distances at *z* ~1**
- **Type Ia supernovae**
- **SNe Ia and cosmology**
- **Results from the Supernova Cosmology Project, the High z Supernova Search, and the HST**
- **Conclusions**

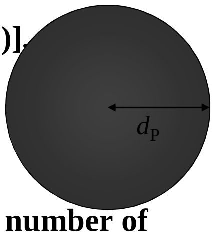
What is distance?

- **Proper distance = integral of RW metric from (r, *t*) to (r', *t*), i.e. distance with d*t* = 0**
  - **we can't actually measure this**
- **How do we measure distance?**
  - **look at apparent brightness of "standard candle"**
    - luminosity distance
  - **look at angular size of "standard ruler"**
    - angular diameter distance



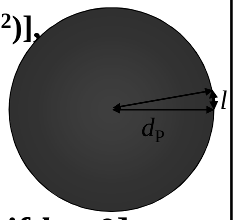
Luminosity distance

- **Luminosity distance d_L is defined by $f = L/4\pi d_L^2$**
  - **consider luminosity L spread out over surface of sphere of proper radius d_P**
  - **$ds^2 = -c^2dt^2 + a^2(t)[dr^2 + x(r)^2(d\theta^2 + \sin^2\theta\, d\phi^2)]$, so area of sphere $A_P = 4\pi x^2$ (now)**
    - $x = R\sin(r/R)$, $k > 0$
    - $x = r$, $k = 0$
    - $x = R\sinh(r/R)$, $k < 0$
  - **also redshift reduces energy per photon and number of photons received per unit time, each by factor (1+z)**
  - **Hence $f = L/4\pi x^2(1+z)^2$**
- **Result: $d_L = x(r)\,(1+z)$ [= $d_P\,(1+z)$ if $k = 0$]**

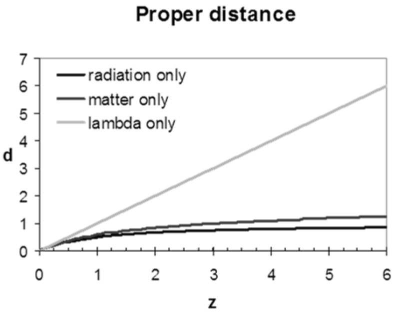


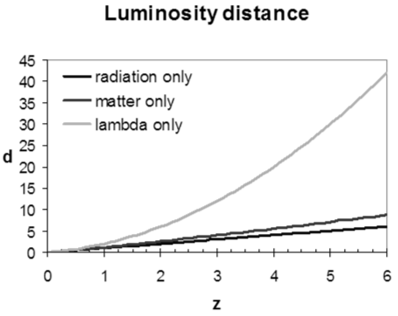
Angular diameter distance

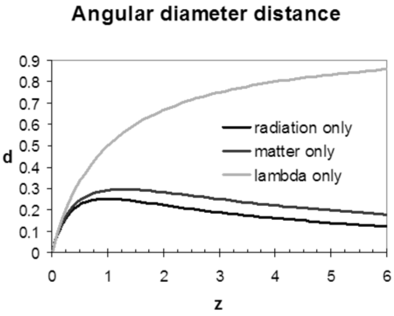
- **Angular diameter distance d_A is defined by $\delta\theta = l/d_A$**
- **consider object of length l viewed at distance d_P**
  - **$ds^2 = -c^2dt^2 + a^2(t)[dr^2 + x(r)^2(d\theta^2 + \sin^2\theta\, d\phi^2)]$, so $l = a(t_e)\,x(r)\,\delta\theta = x(r)\delta\theta/(1+z)$**
    - $x = R\sin(r/R)$, $k > 0$
    - $x = r$, $k = 0$
    - $x = R\sinh(r/R)$, $k < 0$
- **Result: $d_A = d_L/(1+z)^2$ [= $d_P/(1+z) = d_P(t_e)$ if $k = 0$]**
  - **the angular diameter distance is the proper distance at the time the light was emitted**



Distances at large z (k=0)

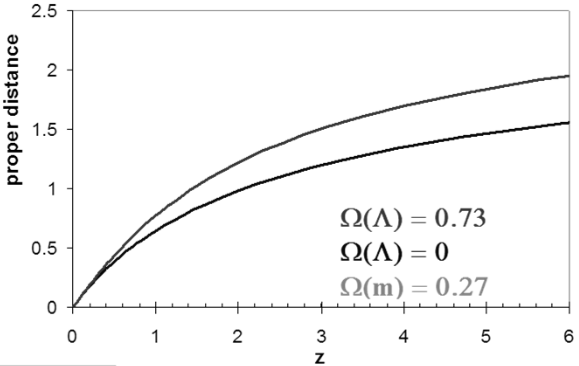




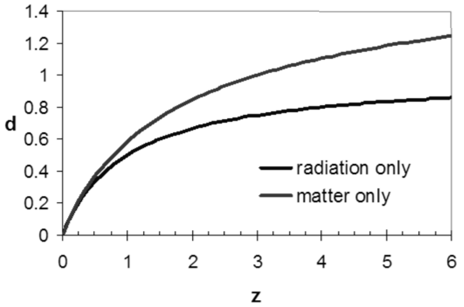


The Hubble diagram carries information about contributions to Ω, but only if we can use *z* > ~½

Distances at large z (k≠0)



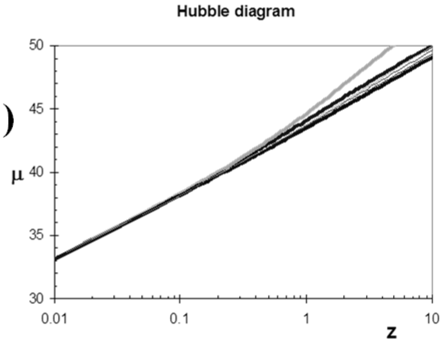
Proper distance



The Hubble diagram can also carry information about *k* – but in general the solution for *k*, Ω_m and Ω_Λ is not unique.

Hubble plot at large z

- **Observable for a standard candle is μ = 5 log(*d*/10 pc)**
  - ***d* here is obviously luminosity distance**
  - **modifying H_0 just adds/subtracts constant offset**
- **For small *z*, Hubble's law is *cz* = H_0d, i.e. μ = 5(log *z* + log(c/H_0) – 1)**
  - **cosmological parameters seen in deviation from linearity at large *z***



Parametrisation

- **Expand *a*(*t*) in Taylor series**

$$a(t) = a(t_0) + \dot a(t_0)(t - t_0) + \tfrac{1}{2}\ddot a(t_0)(t - t_0)^2$$

and divide by *a*(t_0)

$$a(t) = 1 + H_0(t - t_0) - \tfrac{1}{2}q_0H_0^2(t - t_0)^2$$

$$\boxed{q_0 = -\frac{\ddot a a}{\dot a^2}}$$

$$\frac{1}{a(t)} = 1 - H_0(t - t_0) + \tfrac{1}{2}(2 + q_0)H_0^2(t - t_0)^2$$

- **Result (not model dependent)**

$H_0d_P \approx cz[1 - ½(1 + q_0)z]$ **or**
$H_0d_L \approx cz[1 + ½(1 - q_0)z]$ **or**
$H_0d_A \approx cz[1 - ½(3 + q_0)z]$

Expectations

- **What do models predict?**

$$\dot{a}^2 = H_0^2\left(\frac{\Omega_{\mathrm{r}0}}{a^2} + \frac{\Omega_{\mathrm{m}0}}{a} + \left(1-\Omega_{\mathrm{r}0}-\Omega_{\mathrm{m}0}-\Omega_{\Lambda 0}\right) + \Omega_{\Lambda 0}a^2\right)$$

$$\frac{\ddot{a}}{aH_0^2} = -q = -\frac{\Omega_{\mathrm{r}0}}{a^4} - \frac{\Omega_{\mathrm{m}0}}{2a^3} + \Omega_{\Lambda 0}$$

- **For flat universe**
  - **radiation dominated: $q_0 = 1$**
  - **matter dominated: $q_0 = ½$**
  - **lambda dominated: $q_0 = -1$**

For flat universe, both matter and Λ expect that d_L will appear greater when z is large.

Summary so far

- **Distance measurement at large *z* depends on the underlying cosmology you assume, and whether you measure *luminosity distance* or *angular diameter distance***
- **Can parametrise deviation from Hubble's law by deceleration parameter *q***
- **Matter or radiation-dominated universes have *q* > 0; *q* < 0 "smoking gun" for cosmological constant or similar**