Cosmological models

- Cosmological distances
- Single component universes
  - radiation only
  - matter only
  - curvature only
  - Λ only
- Multi component universes

Cosmological distances

- Proper distance between origin and object:
  - $ds^2 = -c^2 dt^2 + a^2(t)[dr^2 + x(r)^2 d\Omega^2]$ (R-W metric)
  - $d_P(t) = a(t) \int dr = a(t)\, r$ (d_P is not a comoving distance)
  - but we know $r = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$ for light emitted at t_e and observed at t_0
  - therefore the proper distance to an object at time t_0 is

$$d_P(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

  - if $t_e = 0$ we call this the horizon distance – it's the furthest we can currently see

Cosmological models

$$\dot{a}(t)^2 = \frac{8\pi G}{3c^2}\left(\frac{\varepsilon_{r0}}{a(t)^2} + \frac{\varepsilon_{m0}}{a(t)}\right) - \frac{kc^2}{R_0^2} + \frac{\Lambda}{3}a(t)^2$$

- **Different components of energy density just add:**
  - **note different *a* dependence**
  - **at small *a*, radiation must dominate**
  - **matter takes over when $a > \varepsilon_{r0}/\varepsilon_{m0}$**
  - **at large *a*, cosmological constant dominates if it exists**
- **Therefore sensible to consider single components**

Radiation only

$$a\,\mathrm{d}a = \sqrt{\frac{8\pi G\varepsilon_{r0}}{3c^2}}\,\mathrm{d}t$$

- **$a = (t/t_0)^{1/2}$ and $\varepsilon_r \propto t^{-2}$**
- **age of universe: $\ln a = ½(\ln t - \ln t_0) \rightarrow H = 1/2t$**
  - **so $t_0 = 1/2H_0$**
- **proper distance:**

$$d_P(t_0) = ct_0^{1/2}\int_{t_e}^{t_0}\frac{\mathrm{d}t}{\sqrt{t}} = 2ct_0\left(1 - \sqrt{\frac{t_e}{t_o}}\right) = \frac{c}{H_0}\left(\frac{z}{1+z}\right)$$

$$d_P(t_e) = d_P(t_0)/(1+z)$$

Matter only

$$\sqrt{a}\ \mathrm{d}a = \sqrt{\frac{8\pi G\varepsilon_{m0}}{3c^2}}\ \mathrm{d}t$$

- $a = (t/t_0)^{2/3}$ **and** $\varepsilon_m \propto t^{-2}$
- **age of universe:** $\ln a = ⅔(\ln t - \ln t_0) \rightarrow H = 2/3t$
  - **so** $t_0 = 2/3H_0$
- **proper distance:**

$$d_P(t_0) = ct_0^{2/3}\int_{t_e}^{t_0}\frac{\mathrm{d}t}{t^{2/3}} = 3ct_0\left(1-\left(\frac{t_e}{t_0}\right)^{1/3}\right) = \frac{2c}{H_0}\left(1-\frac{1}{\sqrt{1+z}}\right)$$

$$d_P(t_e) = d_P(t_0)/(1+z)$$

Curvature only

$$\mathrm{d}a = \sqrt{-\frac{kc^2}{R_0^2}}\ \mathrm{d}t$$

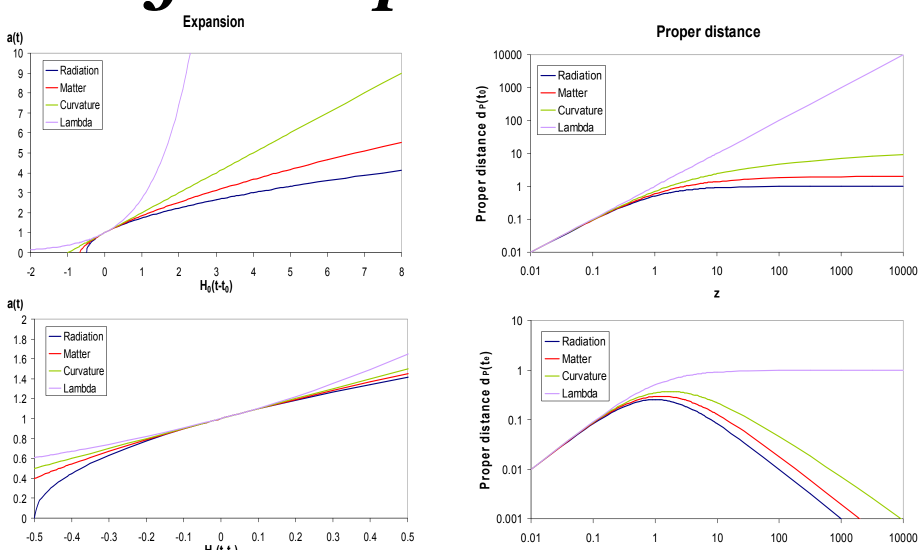
- **if** $k = 0$, $a =$ **constant: flat, static, empty universe**
- **if** $k = -1$, $a \propto t$**: universe expands at constant speed**
  - **Milne model**
  - **age =** $1/H_0$
  - **proper distance** $d_P(t_0) = ct_0 \ln(1+z)$
- $k = +1$ **does not produce a physically viable model**

Λ only

$$\frac{\mathrm{d}a}{a} = \sqrt{\frac{\Lambda}{3}}\,\mathrm{d}t = H_0\,\mathrm{d}t$$

- **$a = \exp[H_0(t - t_0)]$: universe expands exponentially**
  - **de Sitter model**
  - **infinitely old: $a \to 0$ only as $t \to -\infty$**
  - **proper distance $d_P(t_0) = cz/H_0$**
- **this is a "Steady State" universe which always looks the same**

Single component universes

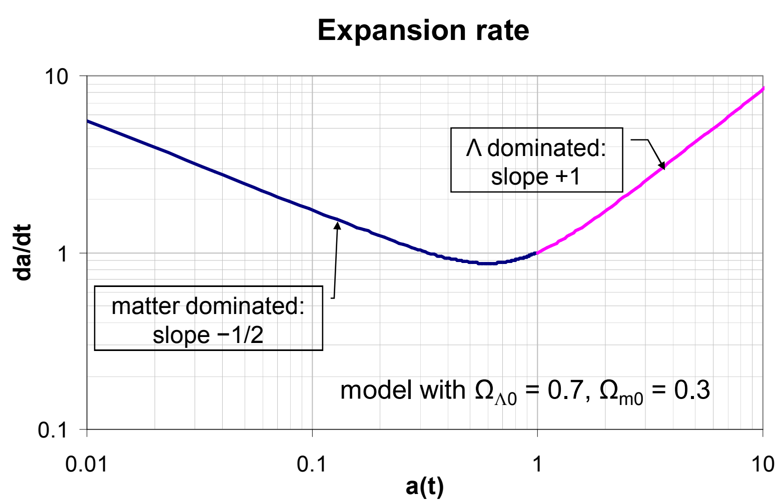


Multi-component universes

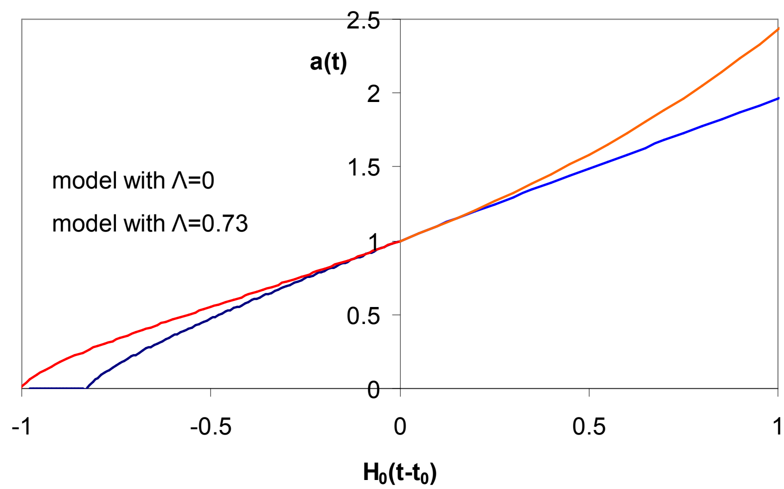
$$\dot{a}(t)^2 = H_0^2\left(\frac{\Omega_{\mathrm{r}0}}{a(t)^2} + \frac{\Omega_{\mathrm{m}0}}{a(t)} + \left(1 - \Omega_{\mathrm{r}0} - \Omega_{\mathrm{m}0} - \Omega_{\Lambda 0}\right) + \Omega_{\Lambda 0}\, a(t)^2\right)$$

- **"This is not a user-friendly integral" (Ryden)**
  - **fortunately at different times different components will dominate**
    - best current values: $\Omega_{m0} = 0.27$, $\Omega_{\Lambda 0} = 0.73$, $\Omega_{r0} = 8.4\times10^{-5}$
    - matter-radiation equality at $a = \Omega_{r0}/\Omega_{m0} = 0.0003$
    - matter-Λ equality at $a = (\Omega_{m0}/\Omega_{\Lambda 0})^{1/3} = 0.72$
  - **at any given time can usually use single-component model**

Example: matter + Λ



Example: matter + Λ



State of Play: theory

- **Friedmann model plus cosmological constant can describe wide variety of behaviour**
  - **expanding, recollapsing or static**
  - **also "bouncing" and "loitering" models**
  - **this technology all available in 1920s**
- **However, models have free parameters**
  - H_0, Ω_{m0}, Ω_{r0}, $\Omega_{\Lambda 0}$
  - **need to determine these to see what model predicts for our universe**