

Relativity

- Principle of relativity
 - ▶ not a new idea!
- Basic concepts of special relativity
 - ▶ ...an idea whose time had come...
- Basic concepts of general relativity
 - ▶ a genuinely new idea
- Implications for cosmology

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Relativity

- “If the Earth moves, why don’t we get left behind?”
- Relativity of motion (Galileo)
 - ▶ velocities are measured relative to given frame
 - ▶ moving observer only sees velocity *difference*
 - ▶ no absolute state of rest (cf. Newton’s first law)
 - ▶ uniformly moving observer *equivalent* to static



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Relativity

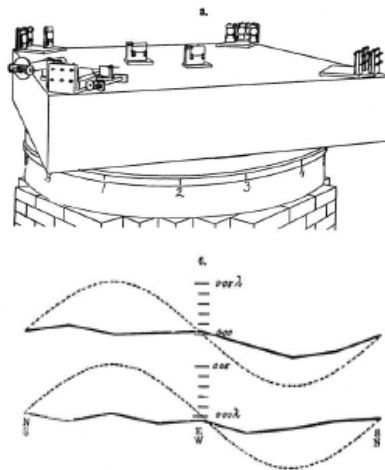
- **Principle of relativity**
 - ▶ **physical laws hold for all observers in inertial frames**
 - ▶ inertial frame = one in rest or uniform motion
 - ▶ **consider observer B moving at v_x relative to A**
 - ▶ $x_B = x_A - v_x t$
 - ▶ $y_B = y_A; z_B = z_A; t_B = t_A$
 - ▶ $V_B = dx_B/dt_B = V_A - v_x$
 - ▶ $a_B = dV_B/dt_B = a_A$
- **Using this**
 - ▶ **Newton's laws of motion**
 - ▶ OK, same acceleration
 - ▶ **Newton's law of gravity**
 - ▶ OK, same acceleration
 - ▶ **Maxwell's equations of electromagnetism**
 - ▶ $c = 1/\sqrt{\mu_0 \epsilon_0}$ – not frame dependent
 - ▶ but $c =$ speed of light – frame dependent
 - ▶ problem!

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Michelson-Morley experiment

- **interferometer measures phase shift between two arms**
 - ▶ if motion of Earth affects value of c , expect time-dependent shift
 - ▶ no significant shift found

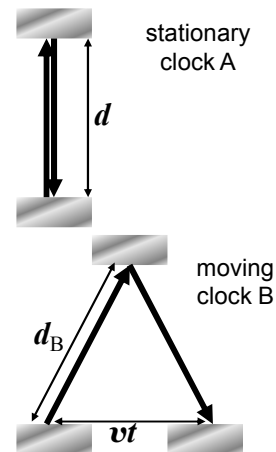


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Basics of special relativity

- Assume speed of light constant in all inertial frames
 - ▶ “Einstein clock” in which light reflects from parallel mirrors
 - ▶ time between clicks $t_A = 2d/c$
 - ▶ time between clicks $t_B = 2d_B/c$
 - ▶ but $d_B = \sqrt{d^2 + \frac{1}{4}v^2t_B^2}$
 - ▶ so $t_A^2 = t_B^2(1 - \beta^2)$ where $\beta = v/c$
 - ▶ moving clock seen to tick more slowly, by factor $\gamma = (1 - \beta^2)^{-1/2}$
 - ▶ note: if we sit on clock B, we see clock A tick more slowly



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Basics of special relativity

- Lorentz transformation
 - ▶ $x_B = \gamma(x_A - \beta ct_A)$; $y_B = y_A$; $z_B = z_A$; $ct_B = \gamma(ct_A - \beta x_A)$
 - ▶ mixes up space and time coordinates \rightarrow spacetime
 - ▶ time dilation: moving clocks tick more slowly
 - ▶ Lorentz contraction: moving object appears shorter
 - ▶ all inertial observers see same speed of light c
 - ▶ spacetime interval $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ same for all inertial observers
 - ▶ same for energy and momentum: $E_B = \gamma(E_A - \beta cp_{xA})$; $cp_{xB} = \gamma(cp_{xA} - \beta E_A)$; $cp_{yB} = cp_{yA}$; $cp_{zB} = cp_{zA}$
 - ▶ interval here is invariant mass $m^2c^4 = E^2 - c^2p^2$

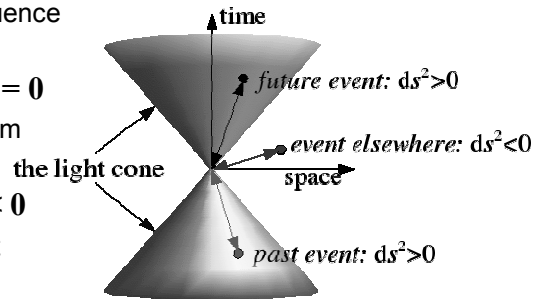
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The light cone

- For any observer, spacetime is divided into:

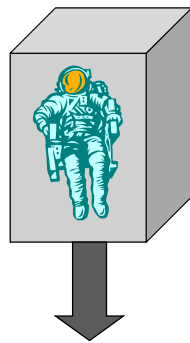
- ▶ the observer's past: $ds^2 > 0, t < 0$
 - ▶ these events can influence observer
- ▶ the observer's future: $ds^2 > 0, t > 0$
 - ▶ observer can influence these events
- ▶ the light cone: $ds^2 = 0$
 - ▶ path of light to/from observer
- ▶ "elsewhere": $ds^2 < 0$
 - ▶ no causal contact



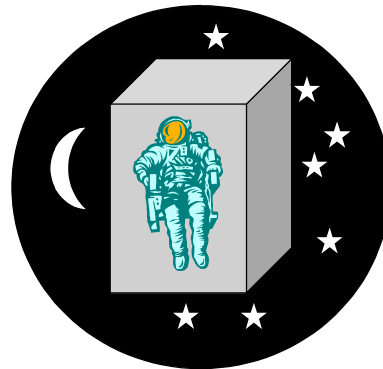
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Basics of general relativity



astronaut in freefall



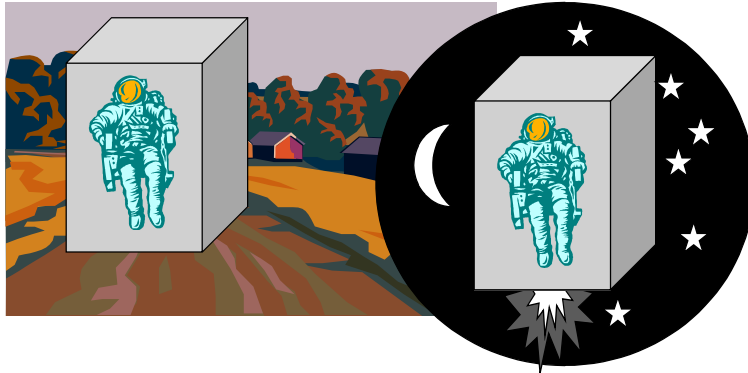
astronaut in inertial frame

frame falling freely in a gravitational field
"looks like" inertial frame

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Basics of general relativity



astronaut under gravity

astronaut in accelerating frame

gravity looks like acceleration
(gravity appears to be a “kinematic force”)

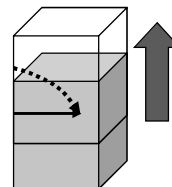
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Basics of general relativity

• (Weak) Principle of Equivalence

- ▶ gravitational acceleration same for all bodies
 - ▶ as with kinematic forces such as centrifugal force
- ▶ gravitational mass \propto inertial mass
 - ▶ experimentally verified to high accuracy
- ▶ gravitational field locally indistinguishable from acceleration
 - ▶ light bends in gravitational field
 - ▶ but light takes shortest possible path between two points (Fermat)
 - ▶ spacetime must be curved by gravity

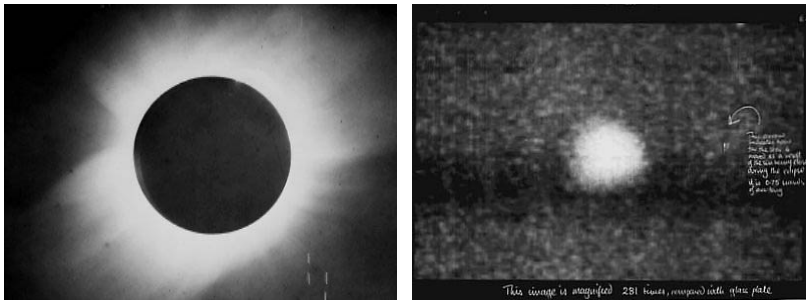


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Light bent by gravity

- First test of general relativity, 1919
 - ▶ Sir Arthur Eddington photographs stars near Sun during total eclipse, Sobral, Brazil
 - ▶ results appear to support Einstein (but large error bars!)

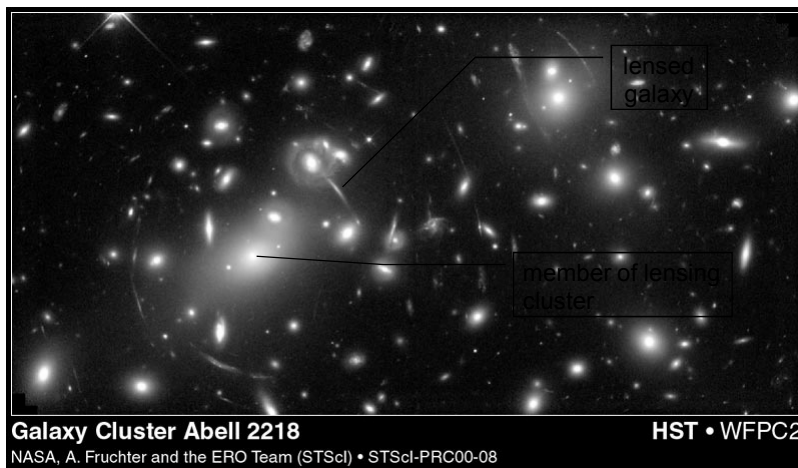


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photos from National Maritime Museum, Greenwich

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Light bent by gravity



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Conclusions

- **If we assume**
 - ▶ **physical laws same for all inertial observers**
 - ▶ i.e. speed of light same for all inertial observers
 - ▶ **gravity behaves like a kinematic (or fictitious) force**
 - ▶ i.e. gravitational mass = inertial mass
- **then we conclude**
 - ▶ **absolute space and time replaced by observer-dependent spacetime**
 - ▶ **light trajectories are bent in gravitational field**
 - ▶ **gravitational field creates a curved spacetime**