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PRIMORDIAL HELIUM ABUNDANCE AND THE PRIMORDIAL FIREBALL. II

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ABSTRACT

This paper contains results of a calculation of the He^3, He^4, and deuterium abundances produced in the early stages of expansion of the Universe. It is assumed in the calculation that the present temperature of the primordial fireball is 3° K, and the abundances are computed for two values of the present mean density of matter in the Universe, and for a range of possible changes in the time scale for expansion of the early Universe.

INTRODUCTION

Subsequent to the suggestion by R. H. Dicke (1964, unpublished) that one ought to search for the residual, thermal radiation left over from the early stages of expansion of the Universe (Dicke, Peebles, Roll, and Wilkinson 1965) a new isotropic microwave background was discovered (Penzias and Wilson 1965). The available information on the spectrum of this radiation (Roll and Wilkinson 1966; Field and Hitchcock 1966; Thaddeus and Clauser 1966) is consistent with the assumption that it is thermal, black-body radiation. If this thermal character is confirmed by further observations, it will be compelling evidence that the new background is the primordial fireball, for it is difficult to see how any process other than thermal relaxation could produce the characteristic thermal spectrum. If we assume that this is established it provides us with the very remarkable opportunity to bring observational evidence to bear on processes which occurred when the Universe was 10^{30} times more dense than it is now (Peebles 1966*a*), when according to general relativity an appreciable amount of helium should have formed.

The purpose of the present article is to describe in greater detail the calculation (Peebles 1966*a*) of the production of helium in the highly contracted Universe. We also show how the amount of element production depends on the time scale for expansion of the Universe through the early, highly contracted phase. This is an important question, since the time scale for expansion could be reduced by the presence of gravitational radiation, or of other new kinds of energy density (Dicke 1966), and since it has been emphasized also that the time scale could be appreciably modified by shear motion in the early Universe (Thorne 1966; Kantowski and Sachs 1966; Hawking and Tayler 1966).

The physical processes of interest here originally were considered in the "big-bang theory," the theory of the formation of the elements in the early, highly contracted Universe (von Weizsäcker 1938; Chandrasekhar and Henrich 1942; Gamow 1948; Hayashi 1950; Alpher, Follin, and Herman 1953). This theory seems to have been generally abandoned with the realization that elements much heavier than helium would not have been formed in any appreciable amount, and with the development of the theory of nucleosynthesis in stars (Burbidge, Burbidge, Fowler, and Hoyle 1957). The great importance here of the primordial fireball, if it exists, is that it provides us with tangible evidence that the universe did pass through a hot, highly contracted phase, so that it becomes of very direct concern to attempt to understand the observable consequences of physical processes that would have occurred in the very early Universe.

In this paper we will assume that the newly discovered microwave background is the primordial fireball—thermal black-body radiation with a temperature of 3° K. The

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1966ApJ...146..542P

numerical results are presented for two assumed values of the mean mass density in the Universe, 7×10^{-31} gm/cm^3 and 1.8×10^{-29} gm/cm^3. The first value is the estimated mass density in ordinary galaxies (Oort 1958; van den Bergh 1961; Kiang 1961), and it is a reasonable lower limit to the total mass density. The second value is the mass density required to close the Universe, assuming a reciprocal Hubble constant $H^{-1} = 1 \times 10^{10}$ yr. This density is a factor of about 3 below the upper limit obtained from measurements of the acceleration parameter (Sandage 1961). We shall confine attention in this paper to the production of nuclei up to helium.

TIME SCALE

In this section we list the equations used to compute the variation of temperature and density with time during the epoch of element formation. The equations are based on an isotropic homogeneous cosmological model, and are equivalent to those previously used by Alpher *et al.* (1953).

According to the simple isotropic homogeneous models the time scale for expansion is given by the equation

$$\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{8\pi G\epsilon}{3c^2}, \tag{1}$$

where the energy density $\epsilon(t)$ satisfies

$$\frac{d\epsilon}{dt} = -\frac{3(\epsilon + P)}{a}\frac{da}{dt}. \tag{2}$$

In these equations P is the pressure of the material and $a(t)$ is the expansion parameter. Here and throughout, all lengths, times, and so on are proper quantities as measured by a comoving observer who uses ordinary instruments. The terms involving the curvature of the model, and the cosmological constant, which usually appear in equation (1) (Tolman 1934) must be negligible at the epoch of element formation, for we know that the Universe has been able to expand out to the present epoch with a general order-of-magnitude agreement between the present values of the left- and right-hand sides of equation (1).

We have assumed that the Universe expanded from an initial temperature of at least 10^{12} ° K. This temperature is high enough to have assured thermal equilibrium, including thermal distributions of neutrino pairs of both kinds, in equilibrium with the radiation and the electron-positron pairs. As the Universe cooled below 10^{10} ° K ($\sim$1 MeV), the electron-positron pairs would have recombined and fed their energy to the radiation. At a temperature of 10^{10} ° K the most effective process for transferring energy to the neutrinos would be electron neutrino scattering. If this process exists, the cross-section would be of the order of 10^{-44} cm^2; and since at a temperature of 10^{10} ° K the characteristic expansion time of the Universe is 1 sec, and the electron-pair density is somewhat less than 10^{32} cm^{-3}, the depth of the Universe for scattering a neutrino is only about 0.03. Thus it is an adequate approximation to suppose that at all temperatures the neutrinos expand according to the simple adiabatic law,

$$T_\nu(t) \propto 1/a(t) . \tag{3}$$

The total energy density in neutrinos of both types, electron and muon, is (Landau and Lifshitz 1958)

$$\epsilon_\nu = \tfrac{7}{4}\sigma T_\nu^4 , \tag{4}$$

where σ is the electromagnetic radiation energy density constant, $\sigma = 7.6 \times 10^{-15}$ erg/cm^3 ° K^4.

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1966ApJ...146..542P

Matter and electromagnetic radiation would remain in thermal equilibrium until long after nucleosynthesis is finished, the variation of the temperature T_e of radiation and matter being given by equation (2) (where we take account of the energy and pressure of the radiation and electron pairs only). We have adopted the free particle approximation for the energy density and pressure of the electron pairs,

$$\epsilon_e = \frac{2}{\pi^2 \hbar^3} \int_0^\infty \frac{p^2 E dp}{e^{E/kT_e} + 1}, \tag{5}$$

$$P_e = \frac{2c^2}{3\pi^2 \hbar^3} \int_0^\infty \frac{p^4 dp}{E(e^{E/kT_e} + 1)}. \tag{6}$$

In these equations $E^2 = p^2c^2 + m_e^2c^4$, where m_e is the mass of an electron. We are assuming here, and will assume below, that the Fermi energies (chemical potentials) of the electrons and neutrinos may be neglected. This is almost implied by the assumption

TABLE 1

NEUTRON PRODUCTION

T_e (10^{10} ° K)	T_e/T_ν	λ (sec^{-1}) (Eq. 17)	$\hat{\lambda}$ (sec^{-1}) (Eq 18)	Time (sec)	X_n (Eq 16)
100..........	1 00	$3\ 60\times10^9$	$3\ 66\times10^9$	0	0 496
60........ ...	1 00	$2\ 79\times10^8$	$2\ 86\times10^8$	$1\ 94\times10^{-4}$	494
30..........	1 00	$8\ 61\times10^6$	$9\ 05\times10^6$	$1\ 129\times10^{-3}$	488
20..........	1 00	1.12×10^6	$1\ 21\times10^6$	$2\ 61\times10^{-3}$	.481
10.........	1 00	$3\ 37\times10^4$	$3\ 91\times10^4$	$1\ 078\times10^{-2}$	.463
6	1 00	$2\ 48\times10^3$	$3\ 19\times10^3$	0 0301	.438
3.........	1 001	$6\ 78\times10^1$	$1\ 12\times10^2$	0 1209	.380
2.........	1 002	7 75	16 4	0 273	.331
1..........	1 008	0 151	0 682	1.103	.241
0.6	1 022	$5\ 70\times10^{-3}$	$7\ 12\times10^{-2}$	3 14	.1969
0 3	1 081	$2\ 12\times10^{-5}$	$3\ 42\times10^{-3}$	13 83	.1719
0 2	1 159	$2\ 08\times10^{-7}$	$5\ 28\times10^{-4}$	35 2	1671
0 1.... .	1 346	$1\ 44\times10^{-11}$	$1\ 95\times10^{-5}$	182 0	.1646
0.03.	1.401	3×10^{-28}	$3\ 3\times10^{-7}$	$2\ 08\times10^3$	0 1640

that the present radiation temperature is 3° K, for with this temperature the leptons would approach degeneracy only if the lepton number were a factor of 10^6 times larger than the nucleon number in the Universe. We believe the most reasonable assumption would be that the lepton number of the Universe is of the same order as the nucleon number, so that the lepton number may be neglected.

The radiation energy density is $\epsilon_r = \sigma T_e^4$. In the assumed model the significant contributions to the total energy density are the radiation density ϵ_r, the energy density ϵ_ν in the two kinds of neutrinos (eq. [4]), and the electron pair energy density ϵ_e (eq. [5]). When the temperature is 10^9 ° K the contribution to the total density due to the nucleons is at most one part in 10^4 of the total density, and this contribution has been neglected.

In the next section we have listed the results (Table 1) of a numerical integration of equations (2), (5), and (6), to obtain the radiation temperature T_e as a function of the expansion parameter, and of equation (1) to obtain the time variation of the expansion parameter.

To take account of the possibility that the time scale for expansion through this highly contracted phase was altered by the presence of gravitational radiation, or of other kinds of neutrinos, or of matter less compressible than radiation, or of shear

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1966ApJ...146..542P

motion, we have computed the element abundances with the assumption that the cosmic time has been scaled by a constant factor S. That is, the time $\hat{t}(T_e)$ for expansion of the model from infinite temperature to temperature T_e is given by the equation

$$\hat{t}(T_e) = St(T_e) , \tag{7}$$

where $t(T_e)$ is the corresponding time for the original simple model. The factor S is a constant. For more general models it may be noted that the amount of element production depends on two factors, the neutron abundance, which is frozen in at a temperature of 1×10^{10} ° K if $S = 1$, and the rate of reactions at the time element formation becomes possible, when $T_e \sim 1 \times 10^9$ ° K. The neutron abundance is given in the next section (Table 2). It is seen that if the expansion time scale were substantially reduced ($S \ll 1$) the neutron abundance would approach a constant value, and element production would depend on the time scale for expansion through the epoch $T_e \sim 1 \times 10^9$ ° K.

NEUTRON PRODUCTION

The production of elements in the early Universe depends on the formation of neutrons due to reactions with the thermal electron and neutrino pairs. The important reactions are

$$p + \bar{\nu} \leftrightarrow n + e^+ , \qquad p + e^- \leftrightarrow n + \nu . \tag{8}$$

We can neglect the reverse of free neutron decay, for we know that when $kT \gtrsim (m_n - m_p)c^2$, the rate of this reaction is that characteristic of free neutron decay, 10^{-3} sec^{-1}, and by the time free neutron decay becomes appreciable the lepton energies are too low to cause further reactions. For the same reason we have neglected the effect of the partial antineutrino degeneracy on the free-neutron decay rate.

The cross-sections for the reactions were computed using the interaction Hamiltonian

$$H_{\mathrm{I}} = \frac{1}{\sqrt{2}} [\bar{p}\gamma_\mu(C_V - C_A\gamma_5)n][\bar{e}\gamma_\mu(1+\gamma_5)\nu] + \text{herm. conjugate}. \tag{9}$$

We are interested in reaction energies of the order of 1 MeV, so we can use the values of the coefficients C_V and C_A obtained from nuclear beta-decay (Källén 1964),

$$C_V = 1.418 \times 10^{-49} \text{ erg cm}^3 , \qquad (C_A/C_V)^2 = 1.39 . \tag{10}$$

The rate for the first reaction (8), from left to right, is

$$\lambda_a = \frac{(C_V{}^2 + 3C_A{}^2)}{2\pi^3\hbar^7 c^3} \int \frac{v_e}{c} \frac{(E_\nu - Q)^2 p_\nu{}^2 dp_\nu}{(e^{E_\nu/kT_\nu} + 1)(1 + e^{-E_e/kT_e})} . \tag{11}$$

This is the reaction rate per proton. In this equation v_e is the velocity of the positron produced in the reaction and E is its kinetic energy (including rest mass), while $E_\nu = p_\nu c$ is the kinetic energy of the incident neutrino. The second term in the denominator is the correction for the partial degeneracy of the positrons. In computing the cross-section and the electron-pair density we have neglected electromagnetic corrections. The threshold for the reaction is $E_\nu = Q + m_e c^2$, where

$$Q = (m_n - m_p)c^2 , \tag{12}$$

and $m_n - m_p$ is the mass difference between a neutron and proton. The integral (11) extends over all neutrino energies greater than the threshold.

The rate of the second reaction (8), from left to right, per proton, is

$$\lambda_b = K \int \frac{(E_e - Q)^2 p_e{}^2 dp_e}{(e^{E_e/kT_e} + 1)(1 + e^{-E_\nu/kT_\nu})} , \tag{13}$$

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1966ApJ...146..542P

the threshold here being $E_e = Q$. The coefficient K is the same as the coefficient appearing in front of the integral in equation (11), p_e is the momentum of the incident electron, $E_e^2 = p_e^2c^2 + m_e^2c^4$, and E_ν is the energy of the neutrino produced in the reaction. The rate of the first reaction (8), from right to left, per neutron is

$$\hat{\lambda}_a = K\int_0^\infty \frac{(E_e+Q)^2 p_e^2 dp_e}{(1+e^{E_e/kT_e})(1+e^{-E_\nu/kT_\nu})}, \tag{14}$$

where the symbols are the same as defined above, and the rate for the second reaction (8), from right to left, per neutron is

$$\hat{\lambda}_b = K\int_0^\infty \frac{v_e}{c}\frac{(E_\nu+Q)^2 p_\nu^2 dp_\nu}{(1+e^{E_\nu/kT_\nu})(1+e^{-E_e/kT_e})}. \tag{15}$$

These formulae are equivalent to the ones given previously by Alpher *et al.* (1953).

The neutron-proton abundance ratio is frozen in at a temperature of 1×10^{10} ° K, when the characteristic expansion time of the Universe is 1 sec. This is well before free neutron decay becomes appreciable and well before element formation can commence (at $T_e \sim 1 \times 10^9$ ° K). Since neutron production and burning thus can be separated in a first approximation, it is useful to compute the neutron abundance, neglecting for the moment neutron decay and element formation. In this case the neutron abundance X_n is given by the equation

$$\frac{dX_n}{dt} = \lambda - (\lambda + \hat{\lambda})X_n, \tag{16}$$

where

$$\lambda = \lambda_a + \lambda_b \tag{17}$$

is the total rate of neutron production per proton (eqs. [11] and [13], and

$$\hat{\lambda} = \lambda_a + \lambda_b \tag{18}$$

is the total rate for destruction of neutrons per neutron (eqs [14] and [15]). The ratio $\lambda/\hat{\lambda}$ would be equal to the Boltzmann factor $e^{Q/kT}$ if the electron and neutrino temperatures were equal.

In Table 1 we have listed some values of the total reaction rates λ and $\hat{\lambda}$ obtained by numerically integrating equations (11) and (13)–(15). Also listed is the ratio T_e/T_ν of the radiation temperature to the neutrino temperature. The neutrino temperature was given by equation (3), and the radiation temperature was found by numerically integrating equations (2), (5), and (6). It will be noted that the ratio T_e/T_ν approaches the value $(\frac{11}{4})^{1/3} = 1.401$, as required by conservation of entropy. Finally, we have given the neutron abundance X_n as a function of time, according to equation (16), with the expansion time scale given by the simple cosmological model ($S = 1$ in eq. [7]). The neutron abundances are in good agreement with the previous results of Alpher *et al.* (1953).

We have also integrated equation (16) assuming other expansion time scales, as defined by the constant S in equation (7). The results are listed in Table 2. The neutron abundance X_n in the table is evaluated at the time the temperature has fallen to $T_e = 1 \times 10^9$ ° K. This is the temperature at which element formation can begin, and at this temperature the value of X_n given by equation (16) is very nearly constant, independent of time.

NUCLEAR REACTIONS

The most important element-forming reactions are

$$n + p \rightarrow d + \gamma\,, \qquad d + d \rightarrow \mathrm{H_e}^3 + n\,, \qquad d + d \rightarrow t + p\,. \tag{19}$$

$$\mathrm{H_e}^3 + n \rightarrow t + p\,, \qquad t + d \rightarrow \mathrm{H_e}^4 + n\,. \tag{20}$$

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1966ApJ...146..542P

At temperatures in excess of 10^{10} ° K the rates of these reactions are very high, so that the element abundances relax to thermal equilibrium. The thermal equilibrium abundance ratios are given by the equations

$$\left(\frac{X_n X_p}{X_d}\right)_{\text{th}} \equiv G_{np} = \tfrac{4}{3}\,\frac{(2\pi m_n k T_e)^{3/2}}{(2\pi\hbar)^3 N}\left(\frac{m_p}{m_d}\right)^{3/2} e^{-B_1/kT_e}, \tag{21}$$

$$\left(\frac{X_d^2}{X_n X_3}\right)_{\text{th}} \equiv G_{n3} = \tfrac{9}{4}\left(\frac{m_d^2}{m_n m_3}\right)^{3/2} e^{-B_2/kT_e}, \tag{22}$$

$$\left(\frac{X_d^2}{X_t X_p}\right)_{\text{th}} \equiv G_{tp} = \tfrac{9}{4}\left(\frac{m_d^2}{m_t m_p}\right)^{3/2} e^{-B_3/kT_e}, \tag{23}$$

$$\left(\frac{X_d X_t}{X_4 X_n}\right)_{\text{th}} \equiv G_{dt} = 3\left(\frac{m_d m_t}{m_4 m_n}\right)^{3/2} e^{-B_4/kT_e}. \tag{24}$$

TABLE 2

NEUTRON ABUNDANCE AT $T_e = 1 \times 10^9$ ° K ACCORDING TO EQ. (16)

Time-Scale Factor S (Eq. 7)	X_n (Eq 16)	Time-Scale Factor S (Eq. 7)	X_n (Eq 16)
3	0 0875	0 01.	0 412
1 . .	1646	0 003 . .	441
0 3 . .	.252	0 001	459
0 1.. . .	319	0 0001 . . .	0 481
0 03 . . .	0 376		

In each of these equations the energy B in the exponential is the Q value of the reaction (König, Mattauch, and Wapstra 1962). On the left-hand sides of these equations in the abundances X represent the number densities of each nuclear species divided by the total nucleon number N per unit volume, so that we have

$$X_n + X_p + 2X_d + 3X_3 + 3X_t + 4X_4 = 1\,. \tag{25}$$

The masses of the nuclei on the right-hand sides of the equations are similarly labeled. We mean by the subscript on the abundance ratios that these are the thermal equilibrium values.

If the present radiation temperature is 3° K, the right-hand side of equation (21) is of the order of 10^{10} when $T_e = 10^{10}$ ° K. This means that at this temperature the deuterium abundance is very small, and by equations (22)–(24) the abundances of heavier nuclei are smaller still. As the Universe expands the temperature decreases and the Boltzmann factor becomes small enough to favor a high helium abundance. The amount of helium actually formed depends on how completely the abundance can relax to the thermal equilibrium value. It should be noted that in taking account only of the relaxation provided by the reactions (19)–(20) we have at least a minimum value of the helium abundance. If any other reaction were found to be important it could only increase the relaxation rate, and so increase the amount of helium formed.

The cross-section for the first reaction (19) varies inversely with relative velocity, so that the reaction rate is determined by the constant factor

$$(\sigma v)_{np} = 4.55 \times 10^{-20}\ \text{cm}^3/\text{sec}. \tag{26}$$

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1966ApJ...146..542P

This cross-section was obtained using detailed balance from the observed cross-section for photodissociation of the deuteron (Hulthen and Sugawara 1957). The cross-section for the first reaction (20) also varies approximately inversely as the relative velocity, and with sufficient accuracy we have for this reaction (Seagrave 1960)

$$(\sigma v)_{n3} = 8.3 \times 10^{-16}\ \mathrm{cm^3/sec}. \tag{27}$$

For the second and third reactions in (19) and the second reaction in (20) it was assumed that the cross-section varies with the reaction energy as

$$\sigma(E_d) = \frac{S}{E_d} e^{-A/\sqrt{(E_d)}}. \tag{28}$$

Here E_d is the energy of the incident deuteron when the other particle involved in the reaction is at rest. This is the conventional Gamow form, but the constants S and A both were chosen to obtain the best fit to the observed cross-section near 100 keV ($\sim 10^9$ ° K). For the second and third reactions in (19) the total cross-section including both reactions (Wenzel and Whaling 1952; Bransden 1960) is adequately represented by the constants

$$S_{dd} = 340\ \mathrm{keV\ barns}, \qquad A_{dd} = 46.2\ \mathrm{keV}^{1/2}, \tag{29}$$

with unity branching ratio to tritium and He^3. For the second reaction in (20) the values of the constants used are

$$S_{dt} = 7.75 \times 10^4\ \mathrm{keV\ barns} \qquad A_{dt} = 49.7\ \mathrm{keV}^{1/2}. \tag{30}$$

This provides a satisfactory fit to the observed cross-section (Fowler and Brolley 1956) when E_d is less than or equal to 100 keV. At higher energies the cross-section given by equation (28) is larger than the observed cross-section, but this is not expected to affect the calculated abundances appreciably.

The cross-section formula (28) was numerically integrated with a Maxwell velocity distribution for the particles to obtain the total reaction rates as functions of the temperature T_e.

In terms of the nuclear abundances defined above (eq. [25]) the production of elements up to helium is determined by the following six equations:

$$\begin{aligned}\frac{dX_n}{dt} = {} & \lambda X_p - (\lambda + \lambda_d) X_n - (\sigma v)_{np} N (X_n X_p - G_{np} X_d) + \tfrac{1}{2} R_{dd} (X_d^2 - G_{n3} X_n X_3) \\ & - (\sigma v)_{n3} N (X_n X_3 - G_{tp}/G_{n3} X_t X_p) + R_{dt} (X_d X_t - G_{dt} X_4 X_n),\end{aligned} \tag{31}$$

$$\begin{aligned}\frac{dX_p}{dt} = {} & -\lambda X_p + (\lambda + \lambda_d) X_n - (\sigma v)_{np} N (X_n X_p - G_{np} X_d) + \tfrac{1}{2} R_{dd} (X_d^2 - G_{tp} X_t X_p) \\ & + (\sigma v)_{n3} N (X_n X_3 - G_{tp}/G_{n3} X_t X_p),\end{aligned} \tag{32}$$

$$\begin{aligned}\frac{dX_d}{dt} = {} & (\sigma v)_{np} N (X_n X_p - G_{np} X_d) - R_{dd} (2X_d^2 - G_{tp} X_t X_p - G_{n3} X_n X_3) \\ & - R_{dt} (X_d X_t - G_{dt} X_4 X_n),\end{aligned} \tag{33}$$

$$\frac{dX_3}{dt} = \tfrac{1}{2} R_{dd} (X_d^2 - G_{n3} X_n X_3) - (\sigma v)_{n3} N (X_n X_3 - G_{tp}/G_{n3} X_t X_p), \tag{34}$$

$$\begin{aligned}\frac{dX_t}{dt} = {} & \tfrac{1}{2} R_{dd} (X_d^2 - G_{tp} X_t X_p) + (\sigma v)_{n3} N (X_n X_3 - G_{tp}/G_{n3} X_t X_p) \\ & - R_{dt} (X_d X_t - G_{dt} X_4 X_n),\end{aligned} \tag{35}$$

$$\frac{dX_4}{dt} = R_{dt} (X_d X_t - G_{dt} X_4 X_n). \tag{36}$$

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1966ApJ...146..542P

In these equations the total rate of the second and third reactions in (19) per unit volume is equal to $R_{dd}(T_e)$ multiplied by the square of the deuteron number density, and similarly $R_{dt}(T_e)$ yields the rate of the second reactions in (20). As mentioned above, these rates were obtained by numerically integrating the cross-section (28), using the constants (29) and (30), with a Maxwell velocity distribution for the particles. The free neutron decay rate is $\lambda_d = \log(2)/11.7$ min (Sosnovsky, Spivak, Prokofiev, Kutikov, and Dobrinin 1959), and λ and $\hat{\lambda}$ were defined by equations (17) and (18).

The equations (31)–(36) were integrated numerically starting from a temperature of 10^{12} ° K. The time variation of the abundances is given in Tables 3 and 4 for the two assumed values of the present mean mass density, in each case the present radiation temperature being 3° K. For these models the final helium abundance by mass was found to be 28 per cent if the present mass density is $\rho_0 = 1.8 \times 10^{-29}$ gm/cm³, and 26 per cent if $\rho_0 = 7 \times 10^{-31}$ gm/cm³. These abundances are slightly smaller than the

TABLE 3

TIME VARIATION OF THE ABUNDANCES BY MASS OF NEUTRONS, DEUTERIUM, HE³, TRITIUM, AND HE⁴, ASSUMING $\rho_0 = 1\ 8 \times 10^{-29}$ GM/CM³, $T_0 = 3°$ K, AND $S = 1$ (EQ. [7])*

T_e (10^{10} ° K)	n	d	He³	t	He⁴
100	0 496	$6\ 3\times10^{-9}$	$1\ 8\times10^{-17}$	$1\ 8\times10^{-17}$	$3\ 3\times10^{-27}$
30	0 488	$1\ 10\times10^{-9}$	$6\ 1\times10^{-19}$	$6\ 0\times10^{-19}$	$3\ 2\times10^{-28}$
10	0 463	$1\ 4\times10^{-10}$	$4\ 0\times10^{-20}$	$3\ 8\times10^{-20}$	$2\ 0\times10^{-29}$
3	0 380	$7\ 1\times10^{-11}$	$1\ 0\times10^{-20}$	$.8\times10^{-20}$	$1\ 7\times10^{-29}$
1	0 240	$6\ 0\times10^{-11}$	$1\ 5\times10^{-19}$	$1\ 1\times10^{-19}$	$2\ 0\times10^{-21}$
0 3	0 1698	$2\ 7\times10^{-9}$	$7\ 5\times10^{-13}$	$2\ 9\times10^{-12}$	$1\ 8\times10^{-13}$
0 16	0 1566	$1\ 4\times10^{-6}$	$1\ 3\times10^{-10}$	$6\ 0\times10^{-9}$	$5\ 8\times10^{-7}$
0 13	0 1500	$3\ 5\times10^{-5}$	$1\ 1\times10^{-9}$	$1\ 6\times10^{-7}$	$2\ 0\times10^{-4}$
0 11	0 1126	$7\ 2\times10^{-4}$	$4\ 3\times10^{-8}$	$3\ 3\times10^{-6}$	0 0605
0 10	0 0119	$5\ 7\times10^{-4}$	$2\ 1\times10^{-7}$	$2\ 7\times10^{-6}$	0 268
0 086	$1\ 1\times10^{-5}$	$2\ 7\times10^{-5}$	$7\ 6\times10^{-7}$	$1\ 3\times10^{-7}$	0 282

* The time is given in Table 1.

TABLE 4

TIME VARIATION OF THE ABUNDANCES BY MASS OF NEUTRONS, DEUTERIUM, HE³, TRITIUM, AND HE⁴, ASSUMING $\rho_0 = 7 \times 10^{-31}$ GM/CM³, $T_0 = 3$ ° K, AND $S = 1$ (EQ. [7])*

T_e (10^{10} ° K)	n	d	He³	t	He⁴
100	0 496	$2\ 4\times10^{-10}$	$2\ 7\times10^{-20}$	$2\ 7\times10^{-20}$	$2\ 0\times10^{-30}$
30	0 488	$4\ 3\times10^{-11}$	9×10^{-22}	9×10^{-22}	$1\ 9\times10^{-32}$
10	0 462	$1\ 0\times10^{-11}$	$6\ 4\times10^{-23}$	$6\ 0\times10^{-23}$	$1\ 2\times10^{-33}$
3	0 380	$2\ 8\times10^{-12}$	$1\ 6\times10^{-23}$	$1\ 3\times10^{-23}$	$1\ 0\times10^{-32}$
1	0 241	$2\ 3\times10^{-12}$	$2\ 2\times10^{-22}$	$1\ 7\times10^{-22}$	$2\ 6\times10^{-27}$
0 3	0 170	$1\ 1\times10^{-10}$	$1\ 2\times10^{-16}$	$4\ 5\times10^{-16}$	$5\ 5\times10^{-19}$
0 12	0 147	$5\ 6\times10^{-6}$	9×10^{-11}	$2\ 3\times10^{-8}$	$1\ 2\times10^{-7}$
0 11	0 143	$3\ 4\times10^{-5}$	$4\ 0\times10^{-10}$	$1\ 6\times10^{-7}$	$4\ 0\times10^{-6}$
0 10	0 137	$3\ 2\times10^{-4}$	$6\ 7\times10^{-9}$	$1\ 5\times10^{-6}$	$2\ 6\times10^{-4}$
0.09	0 116	$3\ 6\times10^{-3}$	7.4×10^{-7}	1.7×10^{-5}	$2\ 7\times10^{-2}$
0 08	$5\ 3\times10^{-3}$	$2\ 8\times10^{-3}$	8×10^{-6}	$1\ 4\times10^{-5}$	0 243
0 07	$1\ 5\times10^{-5}$	3.0×10^{-4}	$2\ 7\times10^{-5}$	$1\ 4\times10^{-6}$	0 256
0 03	7×10^{-8}	$5\ 0\times10^{-5}$	$4\ 9\times10^{-5}$	$2\ 7\times10^{-7}$	0 256

* The time is given in Table 1.

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1966ApJ...146..542P

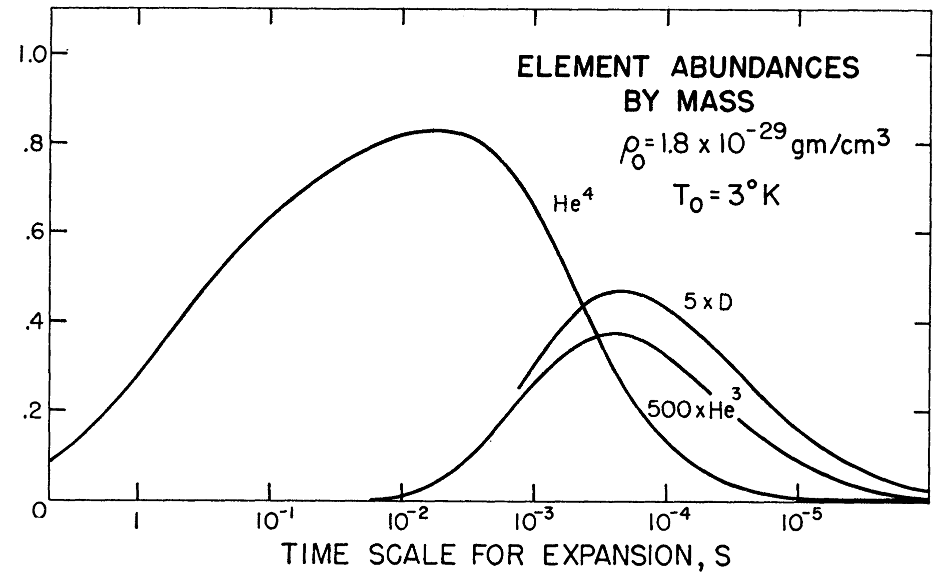

FIG. 1.—Element abundances by mass, produced in the early Universe, assuming present mean density 1.8×10^{-29} gm/cm^3 and fireball radiation temperature 3° K.

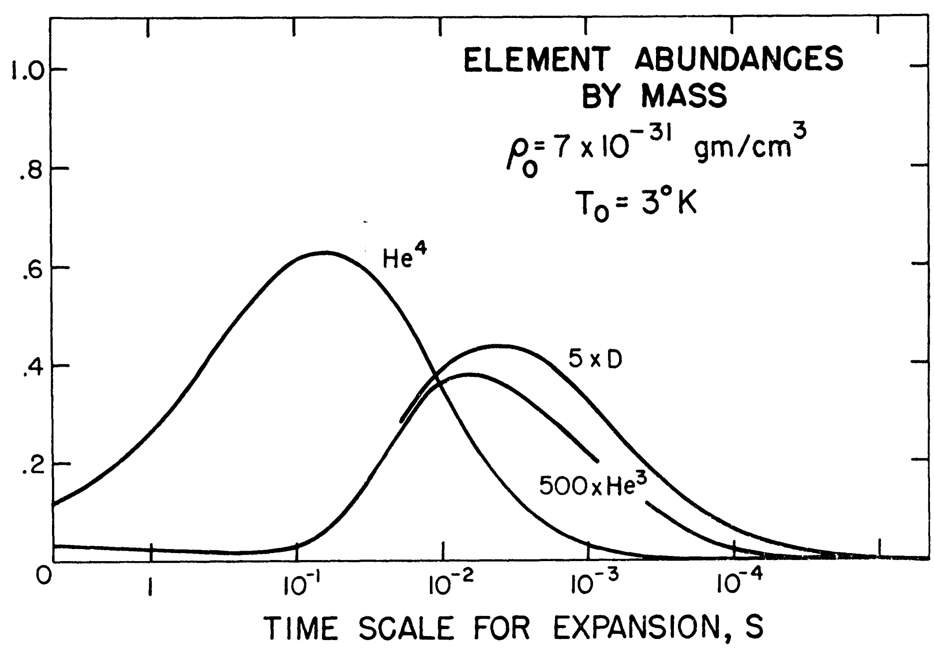

FIG. 2.—Element abundances by mass, produced in the early Universe, assuming present mean density 7×10^{-31} gm/cm^3, and fireball radiation temperature 3° K.

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1966ApJ...146..542P

values previously reported (Peebles 1966*a*), the difference being due to improved numerical accuracy of the integration.

The equations (31)–(36) were integrated also for a range of values of the time-scale factor S in equation (7). The abundances by mass of deuterium, He^3 (including the contribution from the beta-decay of tritium) and He^4 after the completion of nuclear burning are shown in Figures 1 and 2 as functions of S for the two assumed values of the present mass density.

CONCLUSIONS

With the assumption of the simple cosmological model we have concluded that the primordial helium abundance should be in the range 26–28 per cent by mass. Apparently a primordial abundance this high cannot yet be ruled out because we do not have a reliable measure of the helium abundance in very old Population II stars. The helium abundance in the Sun is thought to be 25–30 per cent by mass (Gaustad 1964), but we do not know how much of this helium was produced in earlier generations of stars.

It is important to ask what assumptions one might introduce to arrive at a lower primeval helium abundance. A reasonable upper limit to the possible amount of primordial gravitational radiation would be an energy density equivalent to 1.8×10^{-29} gm/cm^3. This is the density required to close the Universe, with the adopted value of the reciprocal Hubble constant ($H^{-1} = 1 \times 10^{10}$ yr); although it will be noted that, if the Universe were closed by radiation of this amount, it would make the age of the Universe equal to 5×10^9 yr, in serious conflict even with the radioactive-decay ages of the meteorites, 4.5×10^9 yr. (We are here assuming that the cosmological constant is equal to zero.) Gravitational radiation of this maximum amount would decrease the time scale for expansion of the early Universe by a factor of 160. With this factor it is seen from Figure 2 that with the minimum acceptable nucleon density, 7×10^{-31} gm/cm^3, the helium abundance would be about 25 per cent by mass, but the deuterium abundance would be unreasonably high. Thus, gravitational radiation apparently cannot appreciably reduce the primeval abundances, and in fact on the assumed model we must rule out the idea that the Universe could contain primordial gravitational radiation energy amounting to much more than the fireball radiation energy density. This conclusion applies to gravitational radiation already present at the time of element formation.

Next, the basic assumption of a homogeneous, isotropic cosmological model might be questioned (Thorne 1966; Kantowski and Sachs 1966; Hawking and Tayler 1966). It should be noted first that, if the nucleon distribution were irregular but the radiation energy distributed in a homogeneous, isotropic way, it would not appreciably alter the computed element production. The important question is the time scale for cooling the radiation. It is seen from Figure 1 that element production would be effectively eliminated if irregularities could decrease the expansion time scale by a factor of 10^6 below that given by the simple models. A second possibility is that the time be increased by a factor of perhaps 10, thus eliminating the neutrons before any appreciable amount of deuterium can form. It is clearly important, therefore, to examine how one might reasonably alter the expansion time scale through the introduction of irregularities, although we have advanced general reasons (Peebles 1966*a*, *b*) for believing that the early Universe must have been quite isotropic and homogeneous.

Finally, it has been pointed out (Dicke 1966) that the time scale for expansion of the early Universe might be reduced very substantially if there existed a field which exerts a pressure in excess of one third of its energy density. The spatially uniform ϕ-field in the cosmology of Brans and Dicke (1961) has this property, and Dicke (1966) has shown that in this cosmology, assuming appropriate initial conditions, the time scale can be reduced by as much as the factor of 10^6 needed to eliminate element production.

Apparently a hot universe ($T = 3°$ K now) with low primeval abundances of matter

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1966ApJ...146..542P

heavier than hydrogen might be achieved in two ways: by the introduction of very large irregularities in the early Universe or by the introduction of more incompressible matter, such as the scalar field in the Brans-Dicke theory. The first possibility is subject to the philosophical objection that the expanding Universe is in fact unstable, so that we would expect irregularities to grow larger rather than smaller. The difficulty with the second possibility is of course that the presence of such a field has not been experimentally established.

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