## BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1931 November 30

Volume VI.

No. 226.

## COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN.

Some problems concerning the distribution of luminosities and peculiar velocities of extragalactic nebulae, by F. H. Oort.

1. The red-shift in the spectra of the spiral nebulae of which, largely owing to the work done at Mt. Wilson, we know that it increases proportionally with the distance, offers a possibility of deriving relative distances for extragalactic objects. For the farthest nebulae, at least, these distances may prove to be more accurate than any of the estimates which can be made from other criteria. The "peculiar" velocities of the nebulae are supposed to be small. For instance it was found in B.A.N. No. 196 that the average peculiar velocity is probably smaller than  $\pm$  80 km/sec.

These facts suggest the possibility of deriving the distribution of the absolute magnitudes of these objects with the aid of the apparent radial velocities. Considering that the isolated nebulae observed show an average red-shift corresponding to about + 1000 km/sec the velocities might enable us to derive absolute magnitudes with accidental errors less than ± 0<sup>m</sup>·2 as soon as the apparent magnitudes are accurately known.

Knowledge of relative luminosities and especially of the "luminosity curve" of nebulae is important, as the still rather uncertain information at hand has been derived almost entirely from clusters of nebulae and it is desirable to find out whether there is an approximate agreement between this distribution and the luminosity distribution of isolated nebulae in a certain volume of space.

An investigation along this line has recently been undertaken by Hubble and Humason <sup>1</sup>) in connection with their discussion of the wealth of new material obtained in recent years at Mt. Wilson. The distribution of luminosities for 36 nebulae was computed with the aid of radial velocities and this was combined with the distribution of 20 other nebular absolute magnitudes, estimated by means of the brightest stars or variable stars. The combined distribution is shown in Figure 6 of their article (full drawn curve), where

it can be compared with the luminosity curve found from clusters of nebulae (dotted curve).

The result of this comparison is far from satisfactory. For we must note that the isolated nebulae considered have been selected on account of their apparent brightness and that, therefore, their luminosity distribution does not correspond to that of the nebulae in an element of volume. In order to derive this latter distribution, or the "luminosity curve", we must, for each absolute magnitude, divide the frequency found through the volume of the sphere in which nebulae of this absolute magnitude would appear brighter than a fixed apparent magnitude.

If  $\Phi(M)$  represents the luminosity curve, f(m, M) the distribution of the absolute magnitudes of nebulae with apparent magnitudes between  $m - \frac{1}{2}$  and  $m + \frac{1}{2}$ ,  $\rho$  the distance,  $\Delta(\rho)$  the space density, we have

$$f(m, M) = \frac{4\pi}{5 \, Mod} \, \rho^3 \, \Delta(\rho) \, \Phi(M)$$

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$$\Phi(M) = \frac{5 \, Mod}{4 \, \pi \, \Delta(\rho)} \, \log^{(M-m-5)/5} f(m, M).$$

As a first approximation let us assume that the space density is constant \*, then, for each apparent magnitude

$$\Phi(M) = c.10^{3M/5} f(m, M)$$

where c is a constant depending on m only.

From an inspection of the diagram mentioned above it is at once clear that if we apply this factor  $10^{6.6}$  M to the full drawn curve, as we should do, it loses all resemblance to the luminosity curve derived from the clusters.

It seems of interest to make a somewhat more detailed examination of the difference. For this purpose I have computed the quantity  $m-5\log v$  for each isolated nebula for which a radial velocity has been published. The visual apparent magnitudes, m, are

<sup>1)</sup> Astrophysical Journal, 74, 77, 1931; Mt. Wilson Contribution, No. 427.

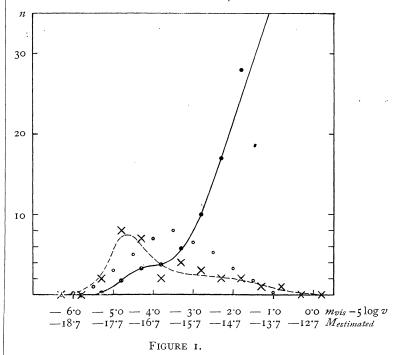
r) Compare Hubble, Astrophysical Journal, 64, 366, 1926; Mt. Wilson Contribution, No. 324.

HOLETSCHEK's values taken from lists by HUBBLE 1), except for the fainter nebulae, for which they were derived from photographic magnitudes determined at Mt. Wilson by subtracting an estimated color-index of  $+ I^{m}$ . o. v Is the velocity of the nebula after correction for a solar velocity of 360 km/sec towards 57° galactic longitude and  $+2^{\circ}$  latitude<sup>2</sup>). The results are summarized in Table 1 showing the numbers of "isolated" nebulae in intervals of half a magnitude. Though it may be seriously doubted whether the various nebulae observed in the Virgo region, from 12h0m to 12h50m right ascension and  $+7^{\circ}$  to  $+20^{\circ}$  declination belong to one definitely co-ordinated system I have not included them in the list of isolated objects used in the present note. The well-established double or multiple nebulae of which more than one component occurred in the list of radial velocities (viz. the Andromeda nebulae, N.G.C. 6658 and 6661, N.G.C. 6702 and 6703) were counted as one object with a magnitude corresponding to the combined light. For the three nearest objects, the Andromeda nebulae, N.G.C. 598 and 6822, all brighter than the tenth magnitude, the radial velocities are too small to be used and I have estimated a quantity equivalent to  $m-5 \log v$  with the aid of HUBBLE's variable star magnitudes and a preliminary expansion coefficient, E, of + 558 km/sec per 10<sup>6</sup> parsecs. Even a considerable change in this value would hardly affect the general appearance of the table. The argument of the table,  $m-5 \log v$ , differs from the absolute magnitude 3) by a constant  $k = 25 - 5 \log E$ , which for the present we may leave undetermined 4). But for this constant shift the numbers in Table I give us the function f(m, M) which should be independent of m if the space density is constant. We notice that the apparently bright nebulae in the second column seem to contain an excess of absolutely bright objects as compared with the distribution of objects fainter than 10<sup>m</sup>·0 (third column). This may, at least partly, be due to the greater uncertainty of the absolute magnitudes for the nearer nebulae, where the peculiar velocities are likely to have a more serious effect than for the distant objects. In the following discussion I have, therefore, omitted the nebulae brighter than 10.0, visual magnitude. The distribution of the 32 fainter nebulae is shown graphically in

Figure I (crosses and dotted curve). This curve is similar to the full drawn curve in Hubble and Humason's Figure 6 (except that right and left have been interchanged and that their magnitudes are photographic). The numbers read from the dotted curve in Figure I are shown under f(M) in the fourth culumn of Table I.

TABLE I.

m 5 log v		$m_{vis}$ < 10.0	$m_{vis} \geq 10.0$	f(M)	Factor	$\Phi\left(M ight)$
6.5 to 6.0 5.5 5.0 4.5 4.0 3.5 3.0 2.5 2.0 1.00.5	0 —6·1 5·6 5·1 4·6 4·1 3·6 3·1 2·6 1·1 0·6 —0·1	0 4 1 3 4 2 1 3 0 1	0 0 2 8 7 2 4 3 2 2 1 1	0.0 0.4 2.5 7.1 6.3 3.7 2.9 2.5 2.2 1.8 0.7	.03 .06 .13 .25 .50 2.00 3.98 7.94 15.8 31.6 63.1 126.	.000 .022 .328 .3.15 .3.77 .5.88 .10.17.28.:41.::44.::38.::
Sum		20	32			



Luminosity curve of isolated nebulae (dots and full drawn curve). Crosses and dotted curve show the luminosity distribution of nebulae fainter than 9.9 vis. mag. and with measured velocities. The open circles represent the luminosity curve of nebulae in clusters (Hubble and Humason). Abscissae indicate values of  $m_{vis} - 5 \log v$  (upper row of numbers) and provisional visual absolute magnitudes (distance 10 parsecs; lower row of numbers). Ordinates are numbers of nebulae in intervals of half a magnitude.

<sup>1)</sup> Astrophysical Journal, 64, 331—337, 1926. The magnitudes given by HUBBLE are HOLETSCHEK's measures reduced to a photometric scale by means of corrections determined by HOPMANN.

<sup>2)</sup> B.A.N., No. 196, 1930; Solution (A).

<sup>3)</sup> Throughout this note absolute magnitudes are magnitudes reduced to a distance of 10 parsecs.

<sup>4)</sup> According to the last section the best estimate for E is + 290 km/sec, corresponding with k=+ 12.7. This value should be subtracted from  $m-5\log v$  in order to reduce it to absolute magnitude. The provisional absolute magnitudes so obtained have been indicated at the bottom of Figure 1.

The factors  $c.10^{\circ 6}$  M by which these numbers should be multiplied in order to derive the luminosity curve are given in the fifth column; the factor has arbitrarily been taken 1.00 for the interval -4.0 to -3.6. The values of  $\Phi(M)$  obtained are in the last column; they have been plotted as dots in Figure 1.

The full drawn luminosity curve which has been traced through these dots deviates markedly from the current ideas about this curve. Towards the fainter absolute magnitudes it becomes increasingly uncertain and the maximum cannot be located, but, if we have any faith in the procedure according to which it has been computed we should accept the fact that there is a difference of at least 2<sup>m</sup>·5 between the average absolute magnitude of nebulae of a given apparent magnitude and the average absolute magnitude of the nebulae in a unit of volume. It is evident that such an amount must make it very difficult to derive a numerical value for the expansion of space by means of clusters of nebulae.

For comparison the distribution curve of the absolute magnitudes of nebulae occurring in clusters, as derived by Hubble and Humason (l. c.) has also been inserted in Figure I (open circles). It has roughly been read off their Figure 6, after the scale of the abscissae had been shifted by the amount  $-1 \cdot 0 + (25 - 5 \log 558) = +10 \cdot 3^{-1}$ ) so as to conform to the scale of  $m_{vis} - 5 \log v$ .

It is clear from Figure 1 that there is no resemblance between the luminosity curve in clusters of nebulae and the curve  $\Phi(M)$  derived above. Before accepting this difference as real let us briefly examine various possible errors. To begin with the  $\Phi$  (M) derived in this section, we must note that the spread in the original f(M)curve will have been too large on account of the accidental errors in the apparent magnitudes as well as on account of peculiar velocities of the nebulae (and of accidental errors in the measurements of the velocities). The accidental error of HOLETSCHEK's visual magnitudes has been computed by LUNDMARK from differences with magnitudes derived photographically 2). He finds a mean error ± 0m·34 for HOLETSCHEK's magnitudes 3). The error is considerable and it would be of the utmost importance to have a homogeneous set of more accurate magnitudes for these nebulae, but the uncertainty does not seem sufficient to explain the great observed spread of the f(M) curve. The uncertainty of the Mt. Wilson

magnitudes used is likely to be considerably smaller and will not have had any influence on f(M). The accidental errors in the measured velocities are not likely, on the average, to exceed  $\pm$  100 km/sec and cannot have had an appreciable influence on the curve for these faint nebulae. Concerning the "peculiar" velocities it has already been pointed out that they appear to be small and of only slight influence. They will, however, be reconsidered in the second section of this note.

As to the cluster curve the lack of published data makes it difficult to judge its reliability. For some clusters, like the so called Virgo cluster, the derivation of the distribution must be exceedingly uncertain, as the distribution of the cluster magnitudes was obtained from a to my mind rather arbitrary dissection of the curve connecting the numbers of nebulae in the region with the apparent magnitude  $^{1}$ ); for others lack of completeness of the fainter nebulae might have played a part. It remains possible, therefore, that the fainter end of this "cluster luminosity curve" would have to undergo some change, though it seems unlikely that this change could be so radical as to make it agree with  $\Phi(M)$ .

2. There is, however, an alternative explanation which is perhaps equally probable as the adoption of a luminosity curve like the one drawn in Figure 1.

In Figure 2 a plot has been made of the quantity  $5 \log v$  against the apparent visual magnitude for all isolated nebulae with positive values of v. It will be seen that the brighter nebulae show a considerable spread around the line drawn under 45°, representing the expansion of space. For the nebulae fainter than 12<sup>m</sup>·5 the spread appears to be considerably smaller; this may well be a chance coincidence 2), but it may also be real and should, therefore, make us careful about accepting at once the conclusion reached in the first section. For, if real, it should be taken as an indication that the true range in absolute magnitude is much smaller than the range shown by the majority of the observed nebulae. This means that the spread of the brighter objects would have to be ascribed mainly to the influence of peculiar velocities of much larger size than assumed at the beginning of this note. The average of  $\pm$  80 km/sec assumed has been inferred from a diagram showing average "residual" velocities

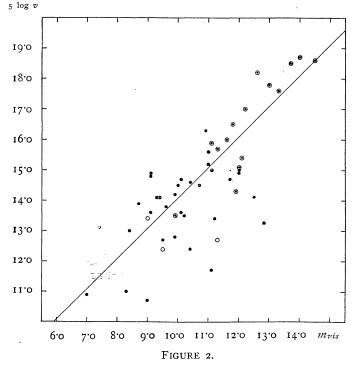
r) 558 km/sec being the coefficient of expansion found by Hubble and Humason.

<sup>&</sup>lt;sup>2)</sup> Arkiv f. Math., Astr. o. Fysik, Bd. 20 A, No. 13, 1927.
<sup>3)</sup> The order of magnitude of the mean error is confirmed by data for 15 stars observed at Mt. Wilson by the extrafocal method. From Harvard data on the Coma Virgo nebulae (Harvard Circular, No. 294) I find a mean error of  $\pm$  om 28 for Holetschek's magnitudes (57 comparisons).

<sup>1)</sup> Harvard Bulletin, No. 865, Figure 9, 1929.

<sup>2)</sup> The nebulae considered are just those observed at Mt. Wilson, but I do not see how they could have been selected so as to conform so closely to the average expansion expected for these magnitudes, unless, of course, the publication of deviating velocities was withheld for the purpose of re-examination. This is, however, very unlikely.

at different distances <sup>1</sup>). I have re-examined the question with the aid of the somewhat different material used in the present note. For this purpose the isolated



Relation between 5 log v and visual magnitude.

The observations made in recent years at Mt. Wilson have been indicated by small circles surrounding the dots. Irregular nebulae are represented by open circles.

nebulae were arranged in groups according to visual magnitude; the average velocity  $\overline{v}$  in each group is shown in the fourth column of Table 2. These numbers have been used to compute the average recession for nebulae of a given apparent magnitude m. Using the numbers of objects in each interval as weights we find -4·I for the total average of  $\overline{m} - 5 \log \overline{v}$ ; the average recession is thus equal to  $10^{\circ 2} (m + 4 \cdot I)$ .

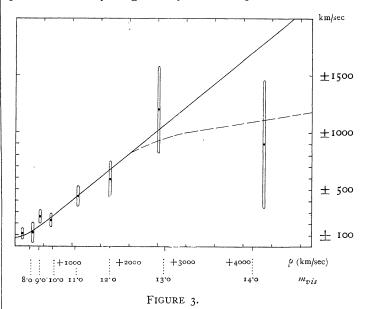
Let us call "residual" the velocity,  $v - 10^{\circ 2}$  ( $m + 4^{\circ 1}$ ), remaining after correction for this average recession and let us compute the average residuals without regard to sign.

TABLE 2.

Interval vis. mag.	n	$\overline{m}_{vis}$	7	average residual	ṁ.e.
5.0 to 7.4 7.5 » 8.4 8.5 » 9.4 9.5 » 10.4 10.5 » 11.4 11.5 » 12.4 12.5 » 13.4 13.5 » 14.5	3 2 8 13 11 8 4	6·5 8·4 9·1 9·9 11·1 12·0 14·1	+ 180 280 620 510 1050 1290 2950 + 5230	± 110 120 260 230 440 590 1200 ± 900	± 50 90 60 60 90 160 380 ± 560

I) B. A. N. No. 196.

The results are shown in Table 2 and Figure 3. In the figure the abscissae are average distances as inferred from the apparent magnitudes. The scale is still unknown, the figures printed below the diagram show average velocities of expansion and apparent magnitudes respectively. The mean error of each average is indicated by a small ellipse. It is quite clear from the diagram that at least from x = 300 to x = 2500 the residuals increase proportionally with the distance, as we should expect them to do if the residuals were entirely due to errors in the correction for recession, i. e. to the spread in the absolute magnitudes. The true average peculiar velocity is given by the extrapolation of this



Residual velocities of nebulae.

Abscissae represent distances. The lower row of numbers shows apparent magnitudes. The upper row is expressed in km/sec and indicates the corresponding velocities of recession:  $10^{o^2} (m + 4^*i).$ 

curve to zero distance and is seen to be probably smaller than  $\pm$  100 km/sec. It is very natural, then, to infer that the whole increase of the residuals with the distance is due to the spread in absolute magnitude but for the fact that, as was shown in the first section, the resulting spread is so much larger than what might perhaps have been expected from other evidence. A similar remark has been made with regard to the previous result about peculiar motion (B.A.N. No. 196), when the mean error of the adopted distances required to explain the slope of the line came out much larger than the mean errors obtained from other considerations.

There seems to be only one alternative possibility, namely that the linear increase of the average residuals is not due to the range in absolute magnitude alone, but also to a real increase in average peculiar velocity. At first sight such a supposition might appear rather

artificial. But considering the apparently great tendency among nebulae to be arranged in agglomerations it would seem quite possible that we too are situated inside or near the border of such an agglomeration. It is not unlikely that the peculiar velocities within an agglomeration are considerably smaller than those of isolated outside nebulae <sup>1</sup>) so that we might expect an increase in average peculiar velocities if we consider more distant objects with a greater admixture of outsiders.

If a hypothesis of this sort were called for to explain an important part of the slope of the straight line in Figure 3 we should have to suppose that most of the nebulae brighter than the 10th magnitude belonged to the agglomeration, the influence of which should be felt down to the 12th or 13th visual magnitude; the outside nebulae should possess an average peculiar radial velocity at least as large as  $\pm$  500 km/sec.

It is necessary to have some more data on radial velocities of faint isolated nabulae before we can dismiss either of the above alternatives. Fortunately not many of these difficult observations would be required; velocities and apparent magnitudes of a few isolated objects of, say, the 15th visual magnitude and selected only according to integrated apparent magnitude would be sufficient. If the solution considered in the first section is true the average "residual" for these objects should lie on the extension of the full drawn line in Figure 3. On the alternative assumption it should lie much lower, on some such line as the dotted curve. The slope of the extension of this latter curve, or, which comes to the same thing, the spread in the values of  $m-5 \log v$  for these faint nebulae, would then determine the real luminosity curve.

If a local agglomeration existed the distribution of nebulae over the sky and the numbers of various apparent magnitudes might be expected to show signs of it. At the present moment, however, these data are not sufficient for any conclusion.

3. The coefficient of expansion. It appears impossible to derive even a rough estimate of the distance of a cluster of nebulae as long as there may be such large and still undeterminable differences between the average luminosity of isolated and cluster nebulae as indicated in the first section. We may be surprised at the accuracy with which Hubble and Humason appear to be able to derive the relative distances of various clusters from apparent magnitudes, agreeing beautifully with the distances derived from radial velocities, but these do not help us in getting absolute

values. Though at present it does not seem possible to make more than a very uncertain guess about the absolute value of space expansion it seems worth while to see how far we can come. For this purpose we have to make an estimate of the mean absolute magnitude,  $M_m$ , of nebulae brighter than a given apparent magnitude (and selected on account of apparent brightness only). If we assume that this mean absolute magnitude is the same as that of fainter nebulae, e.g. those fainter than 9.9 visual magnitude, we can derive the coefficient of expansion. For we have

$$M_m \equiv m - 5 \log v + 5 \log E - 25$$

where the average value of  $m-5\log v$  can be computed from the observed data.

For the different types of nebulae in HUBBLE's system of classification I find the following averages of  $m-5 \log v$  from the nebulae fainter than  $9^{m} \cdot 9$  (the number of nebulae being added between parentheses).

Elliptical 
$$-3.8$$
 (19)  
Sa + SBa  $-4.3$  (9)  
Sb  $-3.6$  (8)  
Sc  $-3.1$  (4)  
[Irregular  $-2.4$  (4)]

The statement made by several investigators that there is no difference between the absolute magnitudes of elliptical nebulae and spirals is confirmed by these numbers. It looks as though there is some difference in absolute magnitude between early and late spirals but the data are too scanty to say that it is real. <sup>1</sup>) The irregular nebulae have been treated in a somewhat different way (see below); they would seem to be still fainter intrinsically than the late spirals.

For the determination of  $M_m$  we may use the intermediate and late type spirals brighter than  $9^m$ 0 according to HOLETSCHEK's corrected visual magnitudes. I have found seven nebulae of this kind in the list observed by HUBBLE and HUMASON. 2) They are tabulated in Table 3:

TABLE 3.

N. G. C.	Туре	$m_{vis}$	$m_S^{}$	$M_{vis}$
224 598 2403 3031 4258 4736 5194	Sb Sc Sc Sb Sb Sb Sc	5.0 7.0 8.7 8.3 8.7 8.4 7.4	16·2 15·6 18·0 18·5 19·5 17·3	— 17.0 — 14.9 — 16.3 — 16.9 — 15.0 — 16.0

<sup>&</sup>lt;sup>r)</sup> According to Hubble and Humason a similar difference has been found by observations of the apparent magnitudes of the brightest stars in Sb and Sc nebulae.

r) On the other hand the expansion of an agglomeration would probably not be different from that of extragalactic space in general.

<sup>2)</sup> Astrophysical Journal, 74, 48, (Table 2).

Only in the two first nebulae variables have been observed; for these the integrated absolute magnitudes (in the last column) would seem to be relatively trustworthy. In all nebulae stars have been observed according to HUBBLE and HUMASON and the average photographic magnitudes for the 4 or 5 brightest stars have been copied in the 4th column of Table 3. HUBBLE and HUMASON advocate the use of these brightest stars for determination of the distance. They give reasons for believing that the absolute magnitude of these brightest objects is a fairly well determined quantity, photographically about equal to  $-6^{M}\cdot 1$ . Not having seen the plates it is difficult from the brief description to form a judgment of the reliability of this method. For one thing it must have been impossible in many cases to discriminate between stars and compact diffuse nebulosities or open and globular star clusters. In the Magellanic Clouds for instance, the absolute magnitudes of these objects merge gradually into those of the brightest stars <sup>1</sup>) and it seems very difficult to determine which limit should apply to the objects observed in a distant extragalactic nebula. The absolute magnitude assumed by HUBBLE and HUMASON may appear to be on the faint side, but adopting it for the present we find the absolute magnitudes of the nebulae as indicated in the last column.

The average absolute magnitude of the two brightest nebulae (derived from variables) is  $-16 \circ$ ; the five nebulae between  $7^{\text{m}} \cdot 4$  and  $8^{\text{m}} \cdot 9$  give  $-15 \cdot 9$ . The average value of  $m-5 \log v$  found from the Sb and Sc nebulae fainter than  $9^{\text{m}} \cdot 9$  was  $-3 \cdot 4$ . Comparing this with the mean absolute magnitude  $M_m$  just derived I obtain E=300 km/sec.

Another, equally uncertain, estimate may be made from the nebulae of the Magellanic Cloud type. Those observed for radial velocity are listed in Table 4, the first two objects being, respectively, the Large and the Small Magellanic Cloud.

TABLE 4.

N. G. C.	$m_{vis}$	v	N. G. C.	$m_{vis}$	v
L. M. C. S. M. C. 3°34	0.2 0.2	— 20 — 60 + 480	4214 4449 6822	9.5 9.5	+ 350 + 300 - 10

If we take the mean absolute magnitude of the two brightest objects, the Magellanic Clouds, as representative of  $M_m$  we find  $M_m = -15^{\circ}2$ . Averaging the magnitudes and velocities for the four remaining objects we obtain  $\overline{m} - 5 \log \overline{v} = -2^{\circ}4$  and E = +280 km/sec. The procedure is clearly very rough and incorrect in more than one respect.

Taking the evidence for what it is worth the best estimate which can be obtained seems to be about + 290 km/sec for the expansion per 10<sup>6</sup> parsecs, but I judge that it can be in error by a factor 1.5, or possibly 2, in each direction.

## Summary.

An attempt has been made to determine the distribution of luminosities of extragalactic nebulae in an element of volume from the observed radial velocities and apparent magnitudes. If the large spread observed in the values of  $m-5\log v$  is accepted as real it must be concluded either that the luminosity curve of isolated nebulae deviates widely from the one found in clusters of nebulae (compare Figure 1), or else that the galactic system is surrounded by an extensive local agglomeration of nebulae, the peculiar velocities of which are much smaller than those of outside nebulae.

The coefficient of space expansion is estimated to be 290 km/sec in a million parsecs; the uncertainty of this coefficient is still very large.

## Positions of minor planets, measured on plates taken by H. VAN GENT,

by G. Pels.

On the plates taken at Johannesburg with the 10-inch Franklin Adams telescope Mr. VAN GENT has identified the minor planets of which the positions are communicated in this bulletin. The plates have been measured at Leiden in the Repsold measuring machine and the

reductions have been performed according to COMRIE's method. (J. B. A. A., vol. 39, p. 203).

The magnitudes have been estimated by Mr. VAN GENT.

r) Compare Shapley and Harvia H. Wilson, Harvard Circulars, Nos. 271 and 275, 1925.