Parallax

Aims

Astronomical knowledge and understanding

This experiment will help you:

- to understand how astronomical distances can be determined using parallax measurements;
- to familiarise yourself with measuring angles in degrees, minutes and seconds, and converting these angles to radians;
- to understand the concept of angular distance and angular separation.

Laboratory Skills

This experiment is designed to develop your skills in:

- reading vernier scales;
- using telescopes
- the value of keeping comprehensive observing logs
- calculating uncertainties and recognising systematic errors.

Introduction

The measurement of distances is one of the great problems in astronomy: how do we measure the distances to points of light in the night sky? Within our own solar system we can measure *relative* distances of the planets quite easily (for example, the ratio of Venus' and Earth's distance from the Sun can be calculated from the maximum angle between Venus and the Sun - the maximum *elongation* of Venus), and *absolute* distances can be measured using the travel time of radar echoes from other planets.

However, for more distant objects, including the nearest stars, the method that was first used to determine distance, and that is still used today is *parallax*: the apparent shift in the position of a nearby object caused by the shift in the position of the observer.

Parallax is easy to demonstrate: hold your arm out in front of you, making a 'thumbs-up' sign. Now close one eye and note the position of your thumb against a background object, such as a picture on the lab wall. Now close that eye and open the other one: note how your thumb appears to shift position against the background. Incidentally, you can find your dominant eye this way: if you start with both eyes open, and close each eye in turn, the position of your thumb will change in one case but not the other. The eye that is open when your thumb's position doesn't shift is your dominant eye.

In this exercise you will measure the distance to a nearby church using the method of parallax. You will measure parallax using both surveyor's theodolites and a telescope equipped with a special eyepiece. The measurements of stellar distances using parallax are exactly the same as the exercise here, though the distances are much greater, and the angles much smaller.

Theory

Principles of Parallax

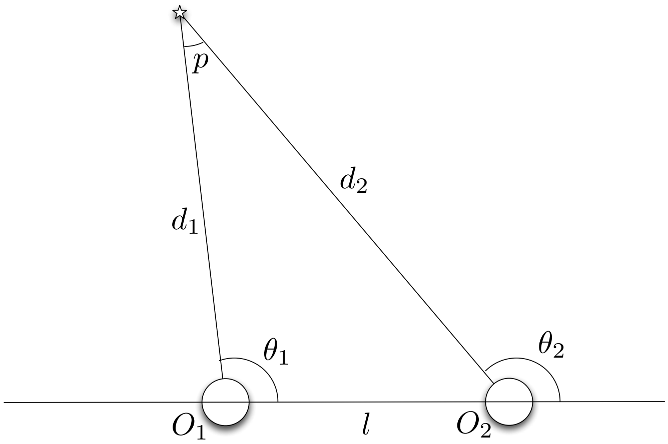

Figure 1: simple parallax geometry

Figure 1 shows the basis of a parallax measurement. An observer moves from O_1 to O_2 and measures the bearing of an object relative to her direction of travel θ_1 and θ_2, respectively. The distance between O_1 and O_2 is *l*. Because the angles of a triangle add up to 180°, we can see that the angle at the apex of the triangle, *p*, is given by $p = \theta_1 - \theta_2$. This angle is the *parallax* of the object.

Using the sine rule, we find

$$\frac{d_2}{\sin\ \theta_1} = \frac{d_1}{\sin\ \theta_2} = \frac{l}{\sin\ p}$$

where we have used the fact that $\sin\theta = \sin(180° - \theta)$.

In astronomy, *l* is small compared to d_1 or d_2, which means that p is a small angle. In this case, *if p is measured in radians*, $\sin p \approx p$. Furthermore $d_1 \approx d_2$, and $\theta_1 \approx \theta_2$. You can satisfy yourself that this is true by imagining what happens as O_1 and O_2 in figure 1 slide closer and closer together. In this case we can drop the subscripts and write

$$d = \frac{l \sin\theta}{p} \qquad (1)$$

where *d* stands for either d_1 or d_2, θ stands for θ_1 or θ_2 and *p* is measured in radians.

More parallax theory

We will use what we have learnt to measure the distance to the Cemetery Chapel on Cemetery Road (CC). Look again at the geometry shown in figure 1. In this diagram, measuring the parallax of CC is easy. However, the angles θ_1 and θ_2 are rather difficult to measure. This is because it is difficult to measure the bearing to the other observing position with great accuracy; unlike the church spire on CC, the theodolites at O_1 and O_2 lack any obvious feature to 'point' at. Since we calculate the parallax, *p*, from the equation $p = \theta_1 - \theta_2$, this means our measured parallax will have large uncertainties.

There is a way to get a more precise measurement of *p*, using the *relative* parallax between CC and another, more distant, church. Since this church will also have a sharp spire, we can measure the angle between this church and CC accurately. In this case we will use St Peter's Church at Greenhill (SP).

Start by assuming St Peter's Church is infinitely far away. In this case the lines of sight to SP are parallel, and the geometry is as shown in figure 2. The angles α_1 and α_2 are the angles between the two churches, and it is possible to show that $p_{CC} = \alpha_1 + \alpha_2$ (ask a demonstrator now if you cannot see why this is).

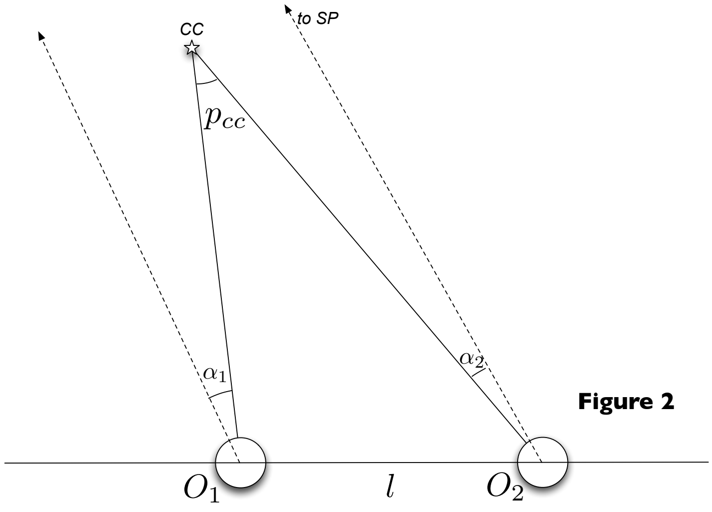

Figure 2

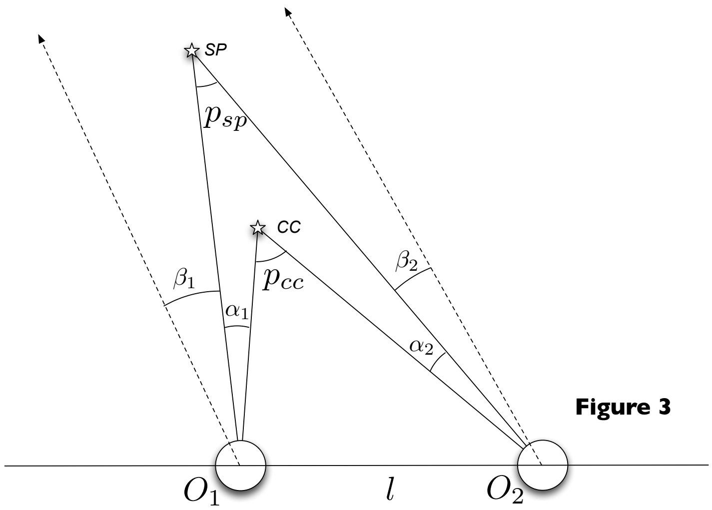

Figure 3

Unfortunately, St Peter's Church is *not* infinitely far away, so the situation is rather more complex than shown in figure 2. The true geometry is shown in figure 3. Once again, α1 and α2 are the angles between the two churches. The angles β1 and β2 are the angles between St Peter's Church and an *imaginary* church which is located at infinity. Thus, $p_{SP} = \beta_1 + \beta_2$. From figure 3, we can show that

$$p_{CC} = \alpha_1 + \alpha_2 + \beta_1 + \beta_2 = \alpha_1 + \alpha_2 + p_{SP} \quad (2)$$

Therefore, if we already have a measurement of the parallax of St Peter's Church, p_{SP}, we can measure the angles between the two churches (α1 and α2) and calculate the parallax of Cemetery Chapel, p_{CC}.

Angular Measurements

There are two systems of units that can be used to measure angles. Astronomers normally use degrees. A full circle is 360°, and each degree is divided into 60 minutes of arc (60'), or *arcminutes*. Each arcminute is in turn divided into 60 *arcseconds* (60"). On the other hand, when using the small angle approximation, and also to perform calculus with angles, we must measure angles in radians. There are 2π radians in a full circle, so 1° ≈ 57.3 radians. Since there are 3600 arcseconds in a degree, we can see that 1 radian = 206265". Be very careful throughout this experiment not to mix different units of measurement, and when using the small angle approximation (including using equation 1) always measure angles in radians.

Vernier Scales

A *vernier scale* is a device which allows fine measurements to be made. The theodolites you will use in this experiment use vernier scales; reading them takes some practice, and it is important that you can do so competently. A vernier scale is a secondary scale which lies alongside the main scale. The simplest vernier scale to understand is a decimal scale; such a scale is shown in figure 4.

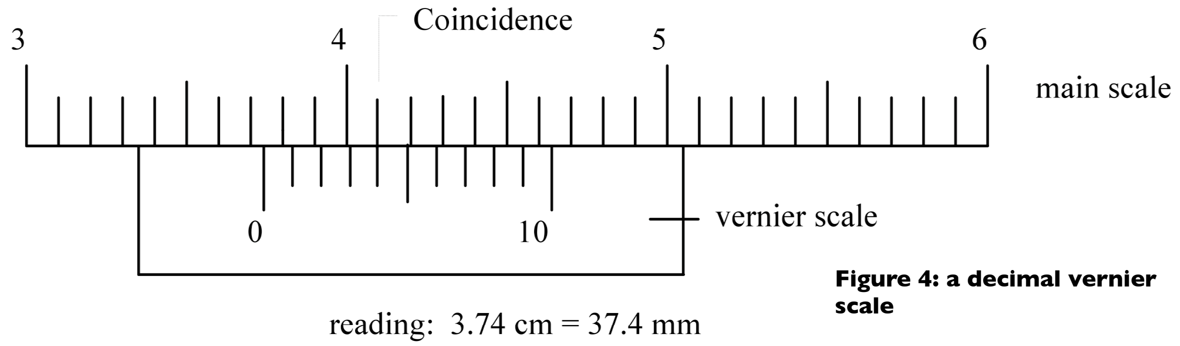

Figure 4: a decimal vernier scale

The position of the 0 on the vernier scale (sometimes marked with an arrow) marks the measured value. In this case it is somewhere between 3.7 and 3.8. The vernier scale allows us to refine this estimate. Each step on the vernier scale corresponds to 1/10th of the smallest subdivision on the main scale. Since the smallest step on the main scale in figure 4 is 0.1, in this case each step on the vernier scale represents a value of 0.01. To read a vernier scale, note the main part of the reading from the main scale; in the example above this is 3.7. The correct value to read off the vernier scale is the line on the vernier scale which lines up exactly with the main scale; in this case the number we should read off the vernier scale is 4. Since each vernier step is 0.01, this tells us we should add 0.04 to the main scale reading, giving a final reading of 3.74.

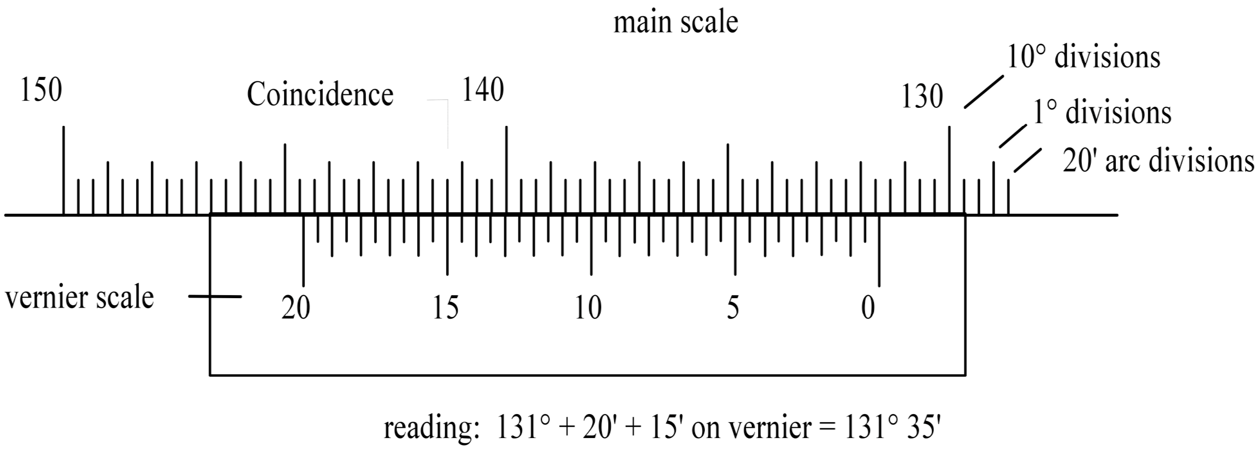

Figure 5: a vernier scale for angles

On the theodolites, the scale is in *degrees*, and the smallest subdivision on the main scale is 20 arcminutes. The vernier scale gives a reading in arcminutes, which should be added to the value read from the main scale. An example is shown in figure 5. In that figure, the main scale gives a reading of 131° 20'. The vernier scale reading is 15' so the final reading is 131° 35'.

How accurately can we read a vernier scale? It depends upon the thickness of the scale divisions. If the lines on the vernier scale are quite thick, several of them may appear to line up exactly. In this case the uncertainty would be a few subdivisions in size. Looking at figure 4, if the lines were quite thick subdivisions 3, 4 and 5 might appear to line up, and the best reading would be 3.74 ± 0.01. If the lines are very fine, the error would be smaller than one subdivision, perhaps 0.002. On figure 4, the line-up looks quite exact, so the best reading would be 3.740 ± 0.002.

What if the none of the markers line up precisely? In this case, the correct reading is based on the marking which is closest to alignment. If the marker on the vernier scale is a little to the right of the marker on the main scale, the measured value should be slightly higher, perhaps 3.745 ± 0.002. Similarly, if the vernier marker is a little to the left, the measured value should be slightly lower.

There are some examples of Vernier scale reading attached to the back of this exercise. Try these, and check your answers with a demonstrator *before going up on the roof.*

Measurements

Locating the Churches

Both churches are more or less due South of the Hicks Building. You can find South by using the Sun as a reference, or by noting that West Street runs West from the city centre to the University, and it's continuation, Glossop Road, continues West for 100m or so in front of the Student's Union. The Cemetery Chapel can be identified by its blackened sandstone spire rising above the trees. St Peters is right on the horizon, and can be identified as a rather squat spire just West of a tree-covered hill. If you are observing from the theodolite nearest the entrance to the roof, St Peter's is slightly East of the Cemetery Chapel. From the other theodolite station, it is slightly West.

Theodolite Measurements

The two theodolite stations are marked by white crosses on the roof. With the aid of the lab technician, set up each theodolite so that it is centred on a white cross. The crosses are 30m apart: estimate the uncertainty in this distance by considering (i) how accurately you think the crosses could have been located in the first place and (ii) how accurately you have centred the theodolites on the crosses.

Level the theodolites using the levelling screws, and lock the scale position using the knob labelled "Lock 1". The other locking screw is labelled "Rotate:Lock" and should be released when making coarse adjustments, e.g. when moving from one church to another, and locked for fine adjustments, which are made by rotating the screw labelled "Rotate:Fine". The technician or lab demonstrator will show you all this when you are on the roof.

Make sure you are happy using the theodolites before you start to take any measurements. The theodolites are easily moved, so take great care not to knock them, or to tread on the flagstones upon which the theodolites rest. Furthermore, get your lab partner to double-check the theodolite is properly aligned immediately before taking your readings.

It will help enormously to keep detailed observing notes in your lab diary. These should include the date and time, weather conditions and any other important information. You will also need to label each observation clearly, so you know which measurements are which when you return to the lab.

From each theodolite station you need to measure the bearings of (1) St Peter's Church (2) the Cemetery Chapel and (3) the other theodolite station. Back in the lab, you will use these bearings to calculate the angles between SP and CC ($\alpha_{1,2}$) and between CC and the theodolite stations ($\theta_{1,2}$).

How will you estimate the uncertainties on these measurements? Each theodolite has two scales, one on each side of the theodolite. At each pointing, you should take a reading from both scales, and keep note of which is which. (Note that the readings on the two scales differ by 180°.) Also, there will usually be two people at each theodolite. Both of you should *independently* take a reading from the theodolite scales, and you should both *independently* line up the theodolite on each target.

This will give you 8 measurements of each bearing (two people × two readings × two pointings). This will be extremely useful for estimating the uncertainties!

Telescope Measurements

Just as we have done for the churches, measuring astronomical parallax relies on precise measurements of the positions of nearby stars, relative to more distant stars. *Astrometry*, the accurate measurement of the positions of astronomical bodies, is now usually done with imaging instruments such as CCD cameras, but this is quite a recent development. Many important studies depend on visual observations using a *micrometer eyepiece*. The telescope set up on a third mark on the roof is equipped with a micrometer eyepiece, and we will use this to take another set of measurements. The telescope station is 5m from nearer theodolite station, and thus 25m from the more distant station.

The telescope is a 5" Celestron reflecting telescope on an equatorial mount. This means it is set up to track the motion of stars in the night sky. Since stars appear to move parallel to the celestial equator, this makes it awkward to move parallel to the horizon. As the technician or demonstrator to show you how to release and adjust the telescope in both directions of rotation, and how to lock it in position.

Use the small finder telescope to locate the two churches. Now look through the main telescope, and make fine adjustments until the church spires are near the centre of the field of view. You will notice that the field consists of a clear central strip surrounded by two translucent regions: this is the effect of the micrometer eyepiece. The eyepiece can be rotated, which adjusts the angle of the clear strip, and has two control knobs which adjust the position and the width of the strip. It also has a digital counter, which reads off the width of the strip.

Rotate the micrometer eyepiece until the clear strip is vertical, and then adjust the two knobs until each church spire is lined up at the edges of the clear strip. Note the counter reading. Unset the micrometer randomly by shifting both knob and let your partner repeat the procedure. Unless you are short of time, do this twice. This will give you four readings; again these multiple readings will help you calculate the uncertainty in your measurements (which is probably dominated by the accuracy with which you can align the church spires and clear edges of the strip).

To use the readings you have just taken, you need to *calibrate* the micrometer eyepiece. Turn the telescope towards the further theodolite station, and focus on the railings. You should see a white plastic ruler slightly to the left of the theodolite. Rotate the micrometer eyepiece so that the clear strip is horizontal, and adjust it so that the strip edges line up with ruler markings 100 mm apart. Record the counter reading, then unset the micrometer and let your partner have a go. As before, repeat this if time allows.

Analysis

Sanity Checking & Uncertainties

From each theodolite station you have bearings to St Peter's Church, the Cemetery Chapel and the other theodolite. You should have eight measurements for each bearing. Use these repeated measurements to estimate the uncertainties on your bearings. What factors contribute to these uncertainties? Whilst estimating your uncertainties, keep an eye out for individual readings which look very different to the others. This could indicate a measurement where you made an error in alignment, or in reading the vernier scale. Such discrepant measurements should be discarded from your analysis - but you must make a note of this in your lab diary!

Theodolite analysis

Using your bearings, and figure 3, you should be able to calculate the angles α_1, α_2, θ_1 and θ_2. For example, the angles $\alpha_{1,2}$ are the differences in bearing between St Peter's Church and the Cemetery Chapel, from each theodolite station.

You now have two ways of calculating the parallax of Cemetery Chapel, using $p_{CC} = \theta_1 - \theta_2$ or from equation (2). Before you can use equation (2) you need to know the parallax of St Peter's Church. The distance to St Peter's Church, as measured by a colleague with a GPS unit is 6.19 km. Use this fact, and equation (1) to calculate the parallax of St Peter's Church, p_{SP}. Don't forget: equation (1) uses the small angle approximation - *all your angles must be in units of radians*! Now use the two methods above to calculate the parallax of Cemetery Chapel, and their uncertainties. Do they agree? If not, why not? Use your two parallaxes together with equation (1) to calculate the distance of CC, with it's uncertainty.

Using an online mapping service, we find that the Ordnance Survey (OS) coordinates of the Hicks Building are (434260, 387240), and those of Cemetery Chapel are (434300, 385890). OS coordinates are measured in metres East (first coordinate) and North (second coordinate) of an arbitrary reference point. Calculate the distance of Cemetery Chapel based on the OS coordinates, with its uncertainty. (The coordinate values quoted above should lead you to suspect the OS coordinates have an uncertainty of ±5 m. What is it about these numbers that suggests this?) Compare your parallax-based distance with the OS-based distance. Do they agree?

Telescope analysis

Calibrate your micrometer eyepiece using the measurements of the ruler. For example, if you measured $n \pm \Delta n$ micrometer units for a length of 100 mm, then 1 micrometer unit = $0.1/36n$ radians, with a fractional error of $\Delta n/n$. Prove these results using geometry and the standard error formulae.

Use this calibration to determine α_T, the angle between SP and CC from the telescope station. The value should be *similar* to the value from the theodolite station nearest the roof entrance. Is it the *same* as that value? Do you expect it to be the same? Explain.

Repeat the parallax and distance calculations from the theodolite analysis, replacing α from the theodolite station nearest the roof entrance with α_T, and making any other corrections necessary. How does your distance compare with the parallax-based distance from the theodolites alone, and with the OS-based value?