HIGH ENERGY PARTICLE ASTROPHYSICS

Acceleration Mechanisms

Acceleration Mechanisms

- There is clear observational evidence that hadrons and electrons are accelerated to extreme energies by astrophysical objects
  - direct evidence: charged cosmic rays
  - indirect evidence: synchrotron emission, inverse Compton emission, TeV photons, evidence of pion production and decay
- Charged particles are accelerated by electric fields, but large-scale permanent electric fields do not exist in nature
  - there are too many charged particles about—space is surprisingly conductive!
- Therefore our source must be magnetic fields
  - stable magnetic fields cannot induce acceleration, but varying magnetic fields can

Acceleration Mechanisms

- Suggested mechanisms for acceleration in astrophysical sources:
  - Fermi second-order acceleration
  - diffusive shock acceleration (DSA)
  - shock drift acceleration (SDA)
  - acceleration by relativistic shocks
  - magnetic reconnection
- Most favoured candidates are DSA and acceleration by relativistic shocks, but probably all of these mechanisms contribute to some degree

ACCELERATION MECHANISMS

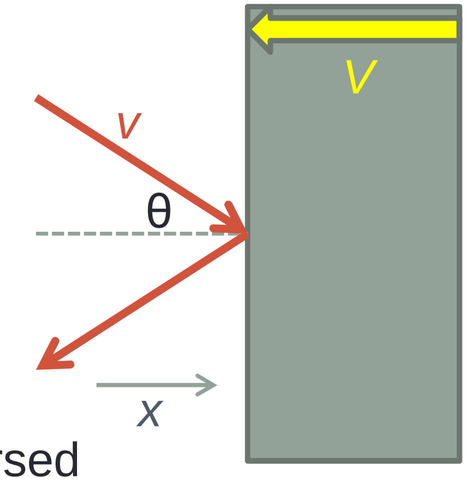
Fermi second-order acceleration

Fermi second-order acceleration

- Original mechanism (Fermi 1949)
  - Particle travelling at speed *v* scatters off magnetic field irregularity travelling at speed *V*
  - In COM frame (= frame of field irregularity) energy of particle is $E_i' = \gamma_V(E_i + \beta_V cp\cos\theta)$ and *x*-momentum is $cp_x' = \gamma_V(cp_x + \beta_V E_i)$
    - After elastic collision, energy unchanged, but p_x reversed
  - Transform back into lab frame: $E_f = \gamma_V\left(E_f' + \beta_V cp_x'\right)$ $= \gamma_V^2(E_i + 2\beta_V cp\cos\theta + E_i\beta_V^2)$
    - Assume *V* ≪ *c* so $\gamma_V^2 = (1-\beta_V^2)^{-1} \simeq 1 + \beta_V^2$ and *v* ~ *c* so $E \simeq cp$
  - Then

$$\frac{\Delta E}{E_i} = 2\beta_V(\beta_V + \cos\theta)$$

neglecting terms higher than β_V^2



Fermi second-order acceleration

- Need to average over $\cos\theta$
  - Cannot neglect relativistic beaming as we are working to order β_V^2, so number density of photons as seen by magnetic field is $\propto \gamma_V(1+\beta_V\cos\theta)$
  - Therefore

$$\langle\cos\theta\rangle = \frac{\int_{-1}^{+1}\cos\theta\,(1+\beta_V\cos\theta)\,\mathrm{d}(\cos\theta)}{\int_{-1}^{+1}(1+\beta_V\cos\theta)\,\mathrm{d}(\cos\theta)} = \frac{\left[\frac{1}{2}\cos^2\theta + \frac{1}{3}\beta_V\cos^3\theta\right]_{-1}^{+1}}{\left[\cos\theta + \frac{1}{2}\beta_V\cos^2\theta\right]_{-1}^{+1}}$$

i.e. $\langle\cos\theta\rangle = \frac{1}{3}\beta_V$

- Substituting this into the equation gives

$$\frac{\Delta E}{E_i} = \frac{8}{3}\beta_V^2$$

Energy spectrum

- If the time between collisions is τ_{coll} and the time to escape is τ_{esc}, we have
  - $\mathrm{d}E/\mathrm{d}t = \alpha E$ where $\alpha = \frac{8}{3}\beta_V^2/\tau_{\text{coll}}$ and
    $$\frac{\mathrm{d}N(E)}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}E}\big(-\alpha E N(E)\big) - \frac{N(E)}{\tau_{\text{esc}}} = -\alpha N(E) - \alpha E\frac{\mathrm{d}N}{\mathrm{d}E} - \frac{N(E)}{\tau_{\text{esc}}}$$
- This will eventually settle down into a steady state in which dN/dt = 0:
  $$E\frac{\mathrm{d}N}{\mathrm{d}E} = -N\left(1 + \frac{1}{\alpha\tau_{\text{esc}}}\right)$$
  - Separating variables gives
    $$\frac{\mathrm{d}N}{N} = -\left(1 + \frac{1}{\alpha\tau_{esc}}\right)\frac{\mathrm{d}E}{E}$$

power law spectrum

$$N(E) \propto E^{-k}$$
$$k = 1 + (\alpha\tau_{\text{esc}})^{-1}$$

Issues with Fermi 2nd order

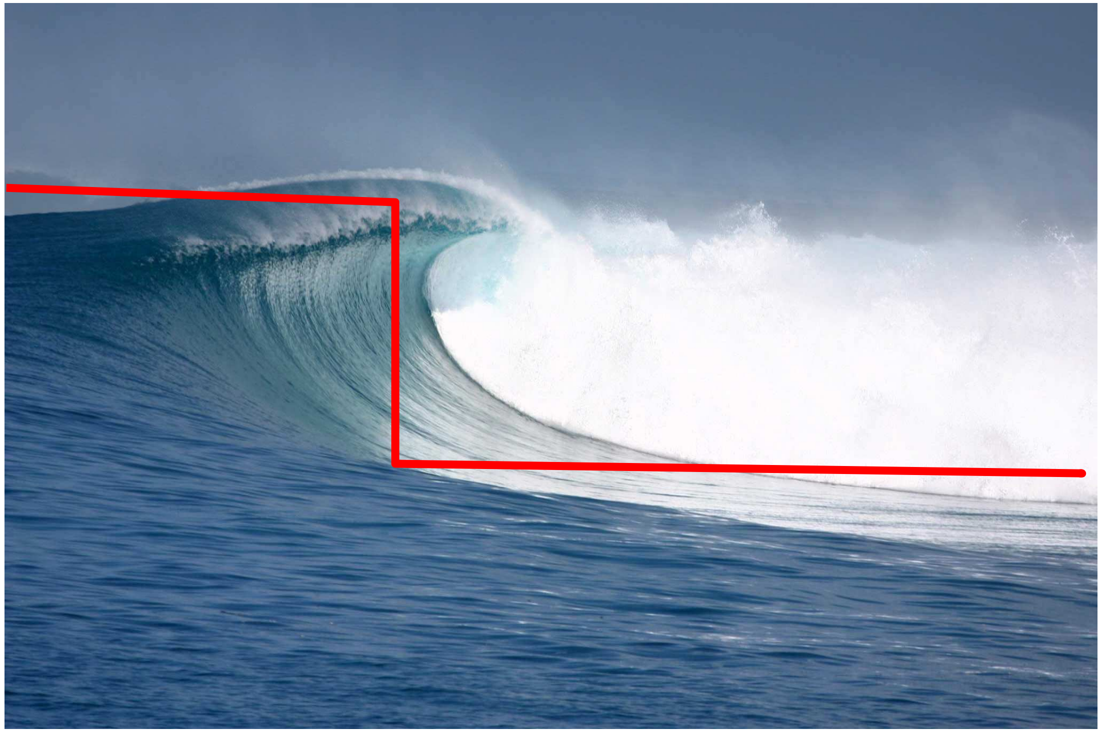
- It's too slow
  - relative velocities of objects in Galactic disc are only 10s of km/s, so fractional energy gain per reflection is only ~10^{-8}
- There is no obvious reason why different sources should yield same spectral index
  - but the fact that the CR spectrum is close to $E^{-2.7}$ over many orders of magnitude suggests that they do
- It is very difficult to get started ("*injection problem*")
  - at low energies, ionisation losses exceed predicted energy gain
  - need seed population to have energies of ~200 MeV or more
    - worse for particles heavier than protons as ionisation loss $\propto z^2$
    - this is far higher than thermal energies
- ***Solution to these problems is acceleration near shocks***

ACCELERATION MECHANISMS

Astrophysical shocks

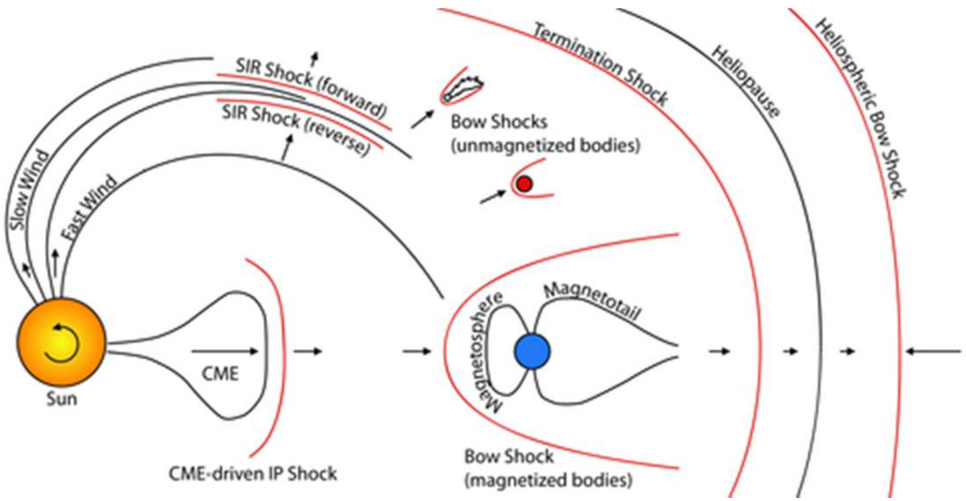
Astrophysical shocks

- Shocks occur when a supersonic flow encounters an obstacle or decelerates to subsonic speed
  - In a situation where a transverse wave would “break”, a longitudinal wave forms a shock front
    - velocity, density and pressure change discontinuously across the shock
- Astrophysical shocks are usually ***collisionless shocks***
  - shock front is much thinner than particle mean free path



Astrophysical shocks

- Many astrophysical objects exhibit shocks
  - several types exist in the solar system and have been studied by spacecraft
    - discontinuities in physical quantities clearly seen



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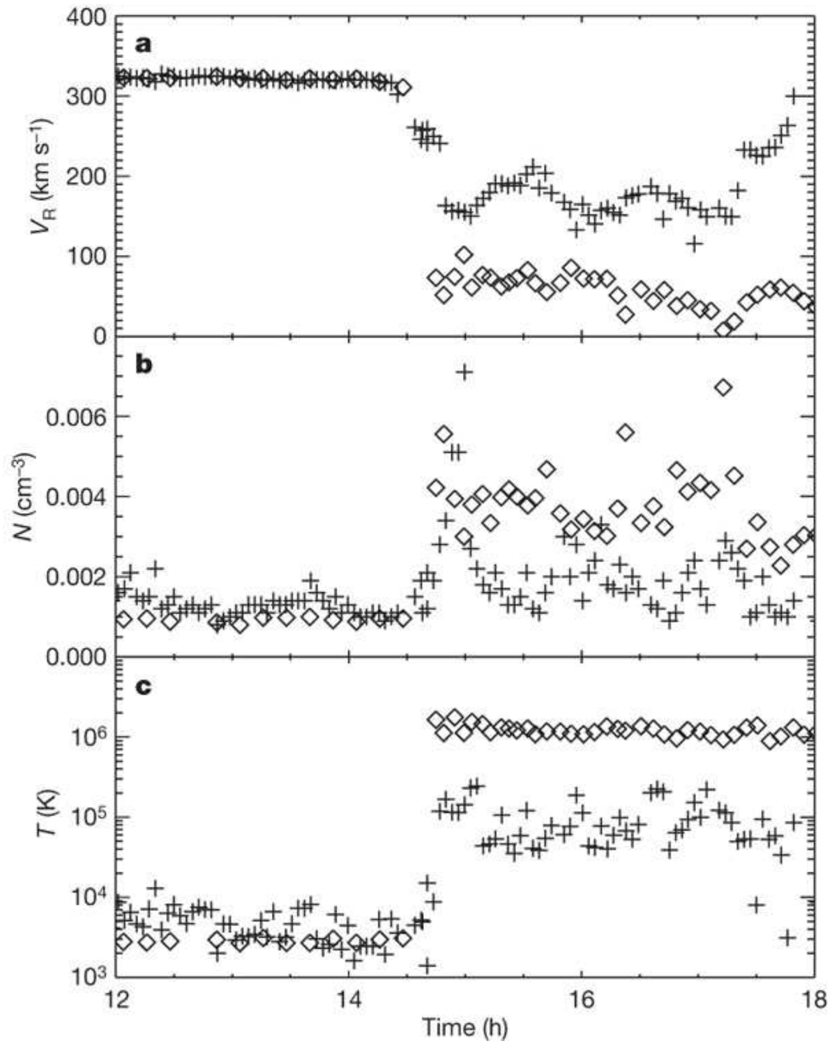
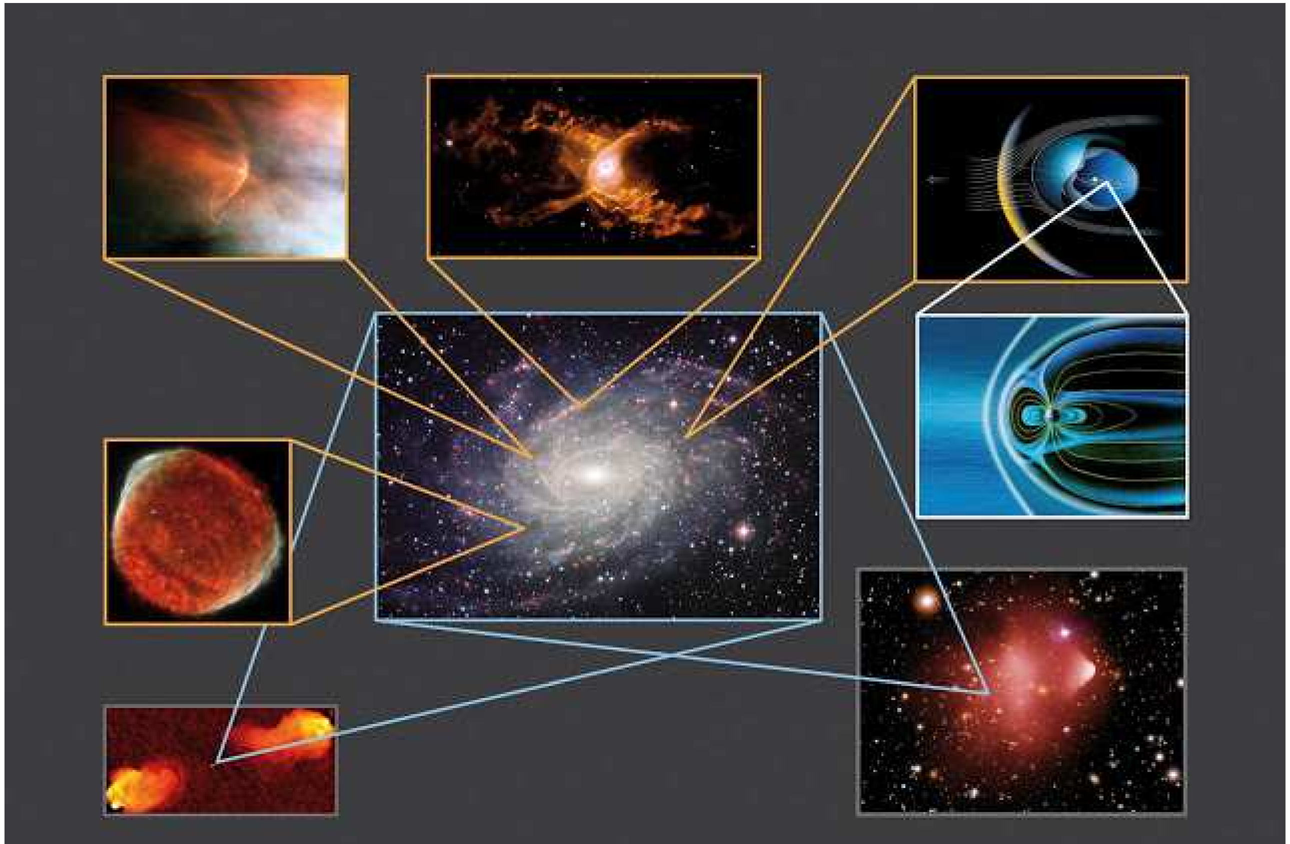
Neptune bow shock ◊ and solar termination shock + observed by Voyager 2



Astrophysical shocks



http://minsex.blogspot.co.uk/2011/11/astrophysical-shock-waves.html

Outside the solar system, shocks are observed as sharp edges in emission, with characteristic shape

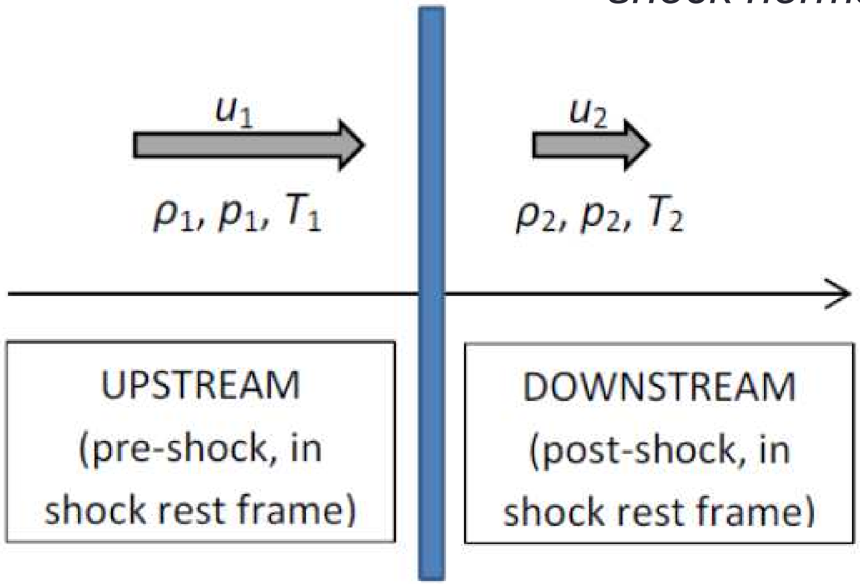
They occur on all scales from planets to clusters of galaxies

Many are clearly associated with acceleration, e.g. seen in synchrotron radiation

Shock jump conditions

plane parallel shock: gas velocity along shock normal

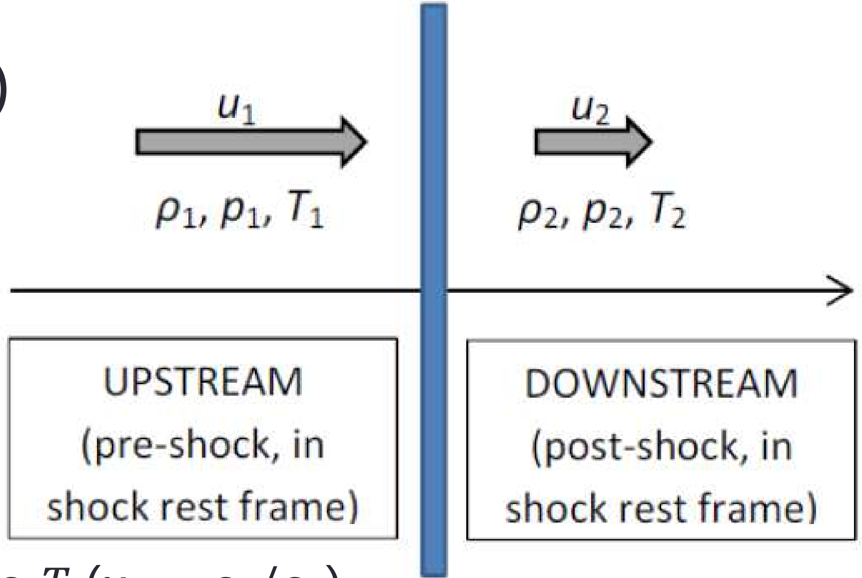
- Consider situation in shock rest frame
  - gas flows into shock with high velocity, low density, low *T*
  - flows out with low velocity, high density, high *T*



- ***Shock jump conditions*** relate pre- and post-shock quantities using conservation laws
  - conservation of mass: $\rho_1 u_1 = \rho_2 u_2$
  - conservation of momentum: $\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$
    - mass of gas crossing shock in time Δt is $\rho_i u_i \Delta t$ (per unit area)
    - change in momentum balanced by change in pressure

Shock jump conditions

- Shock jump conditions (cont.)
  - conservation of energy:
    - energy of gas = internal thermal energy + bulk kinetic energy
    - $\mathcal{E}_i = c_V T_i + \frac{1}{2} u_i^2$ [per unit mass] where c_V is specific heat at constant volume
    - Ideal gas law gives $p = \rho(\gamma_g - 1)c_V T$ $(\gamma_g = c_p/c_V)$



  - Conservation of energy gives $\rho_1 u_1 \mathcal{E}_1 + p_1 u_1 = \rho_2 u_2 \mathcal{E}_2 + p_2 u_2$, i.e.

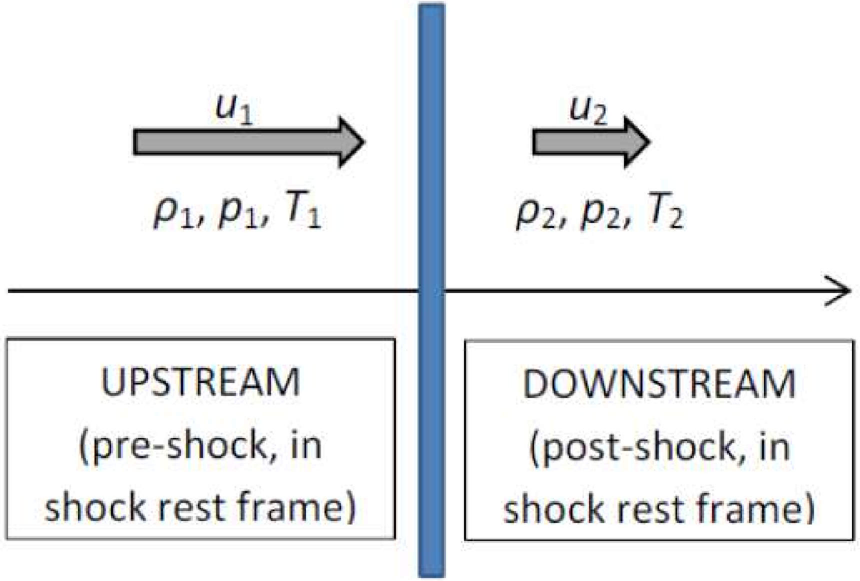
$$p_1 u_1 \left(\frac{\gamma_g}{\gamma_g - 1}\right) + \frac{1}{2}\rho_1 u_1^3 = p_2 u_2 \left(\frac{\gamma_g}{\gamma_g - 1}\right) + \frac{1}{2}\rho_2 u_2^3$$

$$\frac{p_1 \gamma_g}{\rho_1(\gamma_g - 1)} + \frac{1}{2}u_1^2 = \frac{p_2 \gamma_g}{\rho_2(\gamma_g - 1)} + \frac{1}{2}u_2^2$$

this term is the work done, p dV

Shock jump conditions

Putting these three equations together, we have the ***Rankine-Hugoniot conditions*** for a plane-parallel shock:



$$\rho_1 u_1 = \rho_2 u_2$$

$$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$$

$$\frac{p_1 \gamma_g}{\rho_1(\gamma_g - 1)} + \frac{1}{2} u_1^2 = \frac{p_2 \gamma_g}{\rho_2(\gamma_g - 1)} + \frac{1}{2} u_2^2$$

Three equations in three unknowns, therefore soluble

Solution of Rankine-Hugoniot conditions

- Solving these simultaneous equations gives

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma_g + 1)M_1^2}{(\gamma_g - 1)M_1^2 + 2};$$

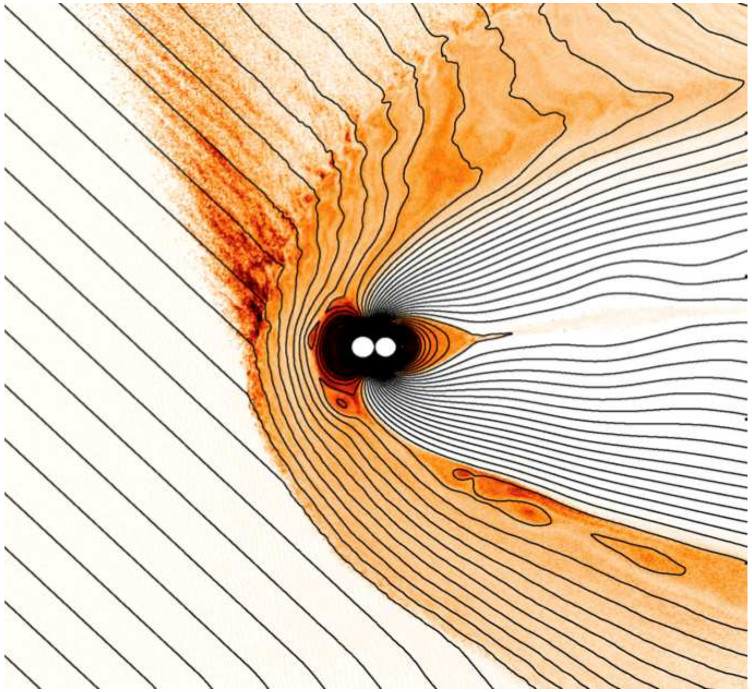
$$\frac{p_2}{p_1} = \frac{2\gamma_g M_1^2 - (\gamma_g - 1)}{\gamma_g + 1}$$

  - where the Mach number $M_1 = \sqrt{\rho_1 u_1^2/(\gamma_g p_1)}$

- For strong shocks, $M_1 \gg 1$, and a nonrelativistic monatomic gas, $\gamma_g = 5/3$, this gives $\rho_2/\rho_1 = 4 = u_1/u_2$
  - reverting to the rest frame of the upstream gas, this means that the downstream gas is accelerated to ¾ of the speed of the shock

Effect of magnetic fields

- Shocks must be associated with magnetic fields to accelerate particles
  - magnetic fields directed close to shock normal have little effect on shock jump conditions—these are ***quasi-parallel shocks***
  - magnetic fields strongly inclined to shock normal contribute an additional “pressure” term—***oblique shocks***
  - shocks in which magnetic field is nearly perpendicular to shock normal are called ***quasi-perpendicular shocks***
  - some shock fronts, e.g. planetary bow shocks, are curved and can have all three geometries at different points



Collisionless shocks and acceleration

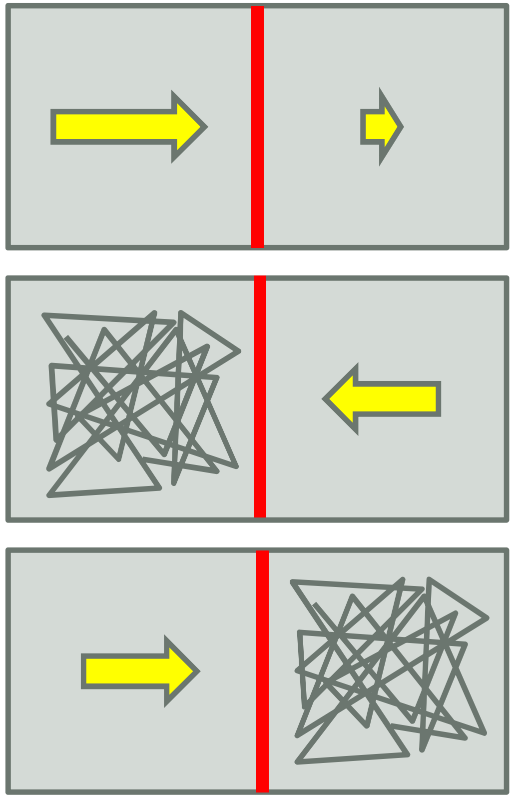
- Because astrophysical shocks are collisionless, population of fast particles can remain out of thermal equilibrium with bulk gas
  - therefore shocks can accelerate particles to high energies, even though they do not accelerate bulk gas that much
- Key parameter is ***criticality*** of shock
  - ***subcritical*** shock can satisfy shock jump conditions while particles remain within (thin) shock front
  - ***supercritical*** shock cannot do this as it is moving too fast
    - therefore it must dissipate energy (generate entropy) some other way
    - it turns out that the natural way to do this is to reflect some of the incoming gas back upstream
    - this is exactly what we want for acceleration
  - boundary between subcritical and supercritical is $\mathcal{M} \simeq 2.76$

ACCELERATION MECHANISMS

Diffusive Shock Acceleration

Diffusive shock acceleration

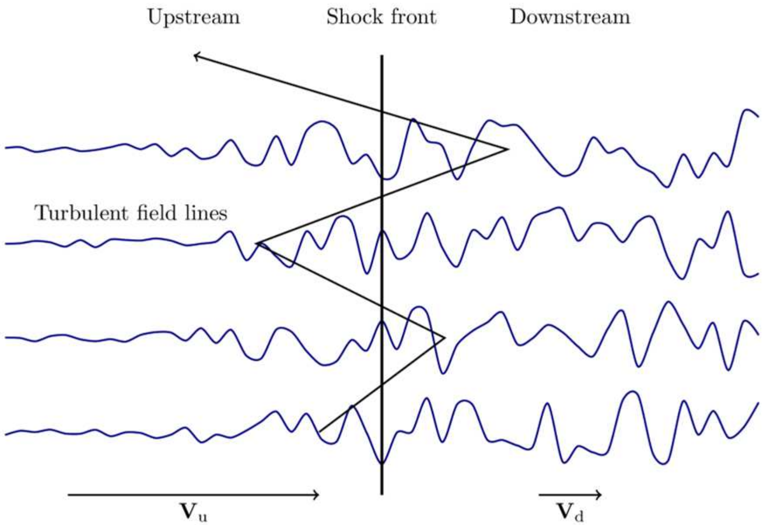
- Plane-parallel shock travelling with speed V
  - in shock rest frame, upstream gas has speed $-V$, downstream gas has speed $-\frac{1}{4}V$
  - in upstream rest frame, downstream gas has speed $\frac{3}{4}V$
  - in downstream rest frame, upstream gas has speed $-\frac{3}{4}V$



- If gas contains a population of fast particles
  - they will scatter elastically until isotropic in the gas rest frame ($\langle \mathbf{v} \rangle = 0$)
  - any particle crossing shock will see gas approaching with speed $\frac{3}{4}V$: collision geometry guaranteed to be favourable

Test particle approach

- Assume fast-particle population does not affect shock
  - Particle with momentum **p** in upstream rest frame has energy $E' = \gamma_U(E + p_x U)$, where *U* = ¾*V*, in downstream rest frame
  - Assume fast particles are ultra-relativistic, *E* ≈ *cp*
    - probability of given particle crossing shock in given time interval is $P(\theta)\mathrm{d}\theta = 2\sin\theta\cos\theta\,\mathrm{d}\theta$
    - if shock is non-relativistic can take $\gamma_U \approx 1$
    - therefore average energy gain is
      $$\left\langle\frac{\Delta E}{E}\right\rangle = \frac{U}{c}\int_0^{\pi/2} 2\cos^2\theta\sin\theta\, d\theta = \frac{2}{3}\frac{U}{c} = \frac{1}{2}\frac{V}{c}$$
    - The same is true for a crossing from downstream to upstream
  - Therefore average energy gain in one return crossing is *V*/*c*
- This is diffusive shock acceleration, or *Fermi first-order acceleration*



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DSA energy spectrum

- After each shock crossing, $E_k = fE_{k-1}$ where $f = 1 + \frac{V}{c}$
- If fast particles have $v \approx c$
  - number crossing shock front per unit area per unit time = ¼Nc (N number density)
  - number advected downstream = ¼NV
  - therefore fraction lost per unit time is V/c
- Hence after k shock crossings, $E_k = f^k E_0$ and $N_k = P^k N_0$ where $P = 1 - \frac{V}{c}$
  - therefore $N(E > E_k) = N_0(E_k/E_0)^{\ln P/\ln f} \simeq N_0(E_k/E_0)^{-1}$
    - since $\ln P \approx -V/c$ and $\ln f \approx +V/c$
  - hence $N(E)\,dE \propto E^{-2}\,dE$, independent of details of shock

Maximum energy from DSA

- Expect a high-energy cut-off because at high energies the magnetic field of the source will not confine the particles
  - For order of magnitude estimate, take Maxwell's equation

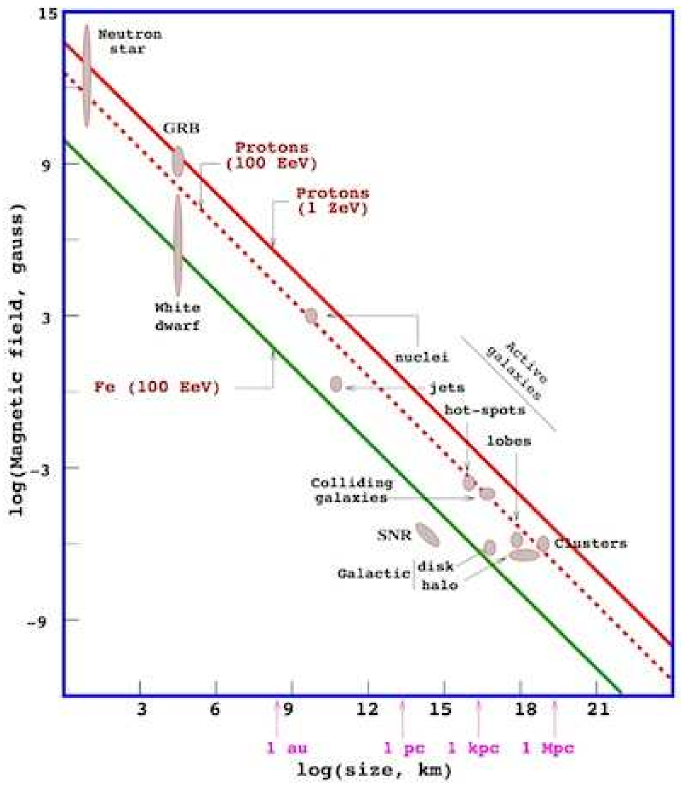
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

  - replace derivatives by divisions

$$\frac{E}{L} \sim \frac{B}{L/V}$$

V speed of shock
L size of source

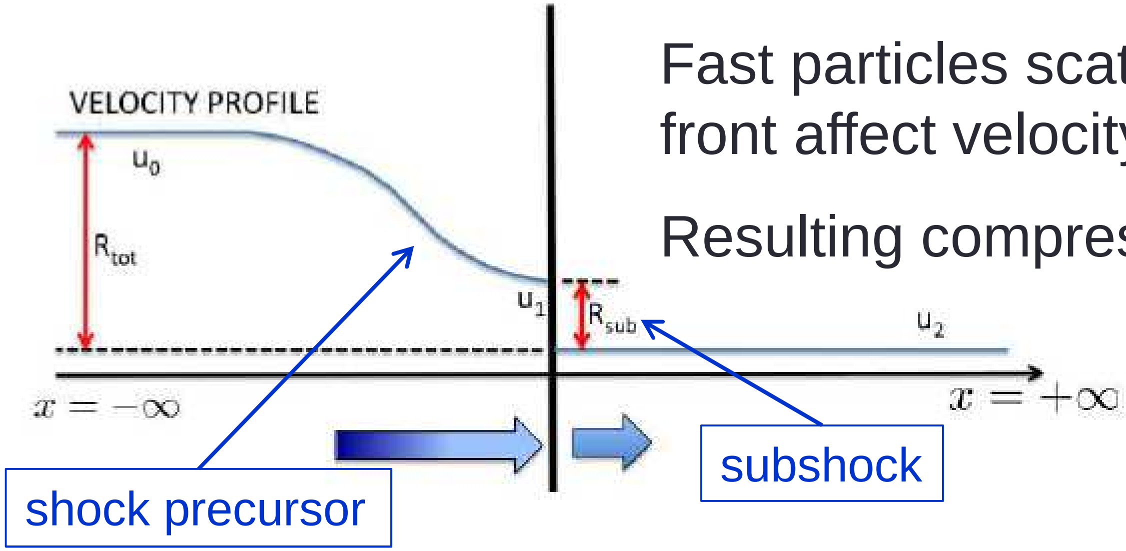
  - then for particle of charge *ze*, $E_{max} \sim zeEL \sim zeBVL$
- This is the basis of the *Hillas plot* of magnetic field against size, used to evaluate potential CR sources



Realistic DSA

- Test particle approach assumes fast particles are negligible as regards their effect on the shock
  - but for supercritical shocks this cannot be right as we know that such shocks need a *significant* reflected flux to conserve energy and momentum across the shock front
  - also, measurements in solar system shocks indicate that acceleration is quite efficient—as much as 20% of the kinetic energy of the bulk gas is used in accelerating high-energy particles
- Therefore the test-particle approach is inadequate and we need a more realistic treatment
  - this is highly non-linear and cannot be done analytically
  - detailed 3D computer simulations are needed
    - 3D because particle distributions are highly anisotropic and turbulence is involved

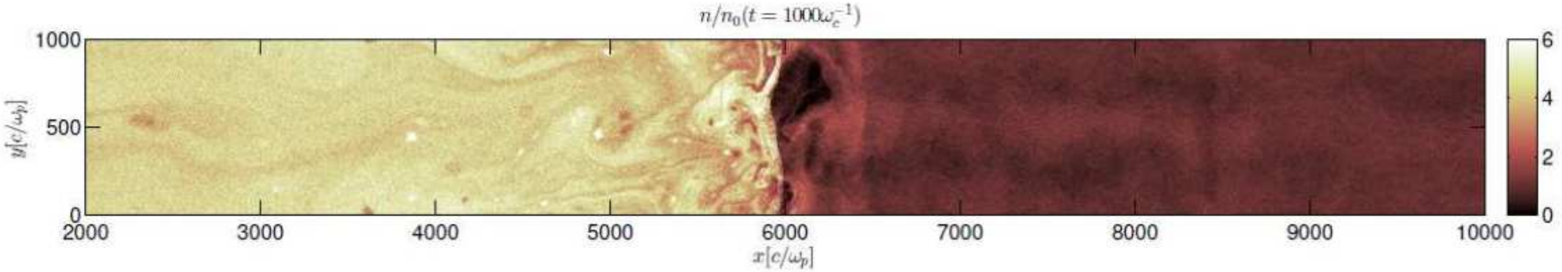
Realistic DSA

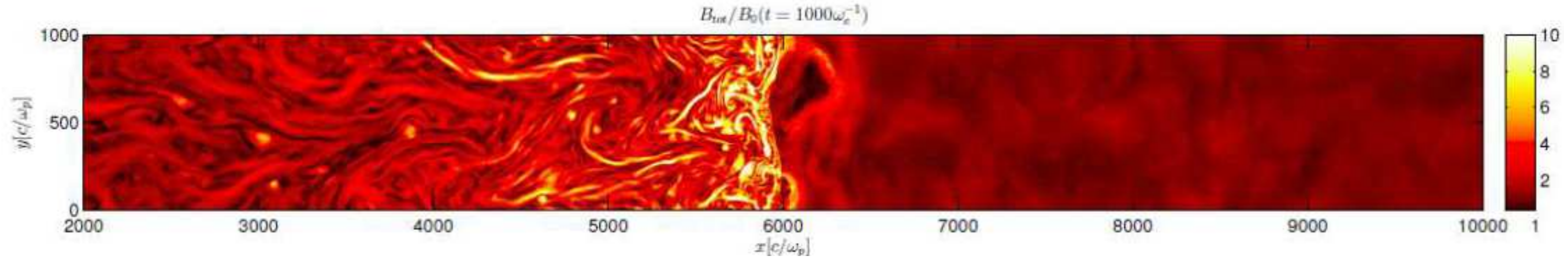


Fast particles scattered back across shock front affect velocity profile of gas

Resulting compression ratio can be >4

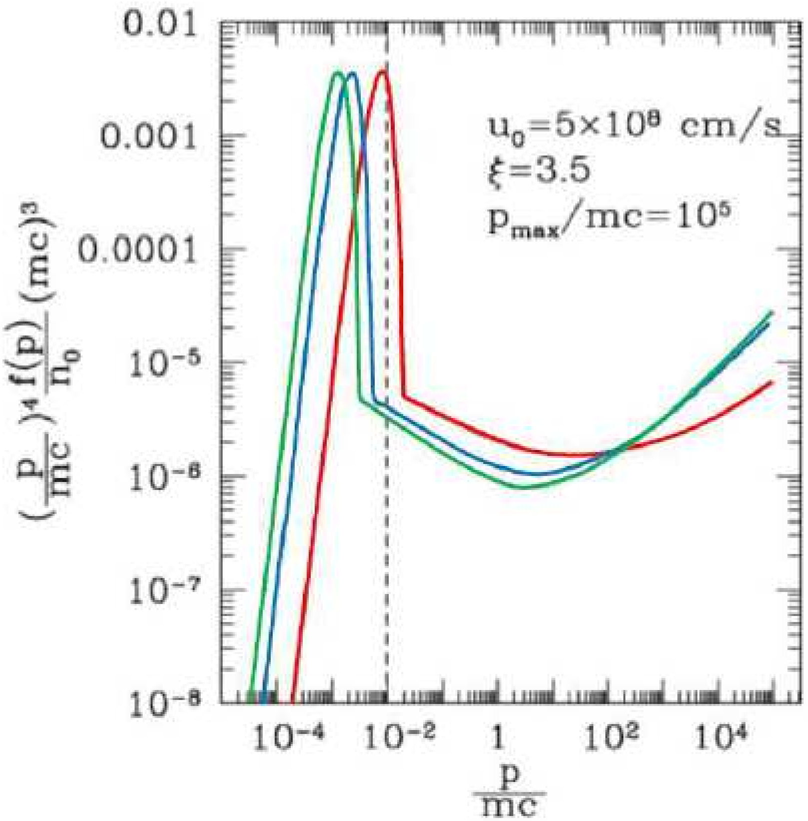
Particles can also generate turbulence via streaming instabilities





Realistic DSA

- Energy spectrum for realistic DSA simulations tends to be concave rather than flat
  - this plot from Blasi is the phase space density $f(p)$ scaled by p^4
  - a $1/E^2$ spectrum would be horizontal on this plot (since $d^3\mathbf{p} = 2\pi p^2\, dp$)
    - the conclusion from this would be that the energy spectrum at high energies should be somewhat less steep than $1/E^2$
    - this is a little unfortunate, since such evidence as there is suggests that if anything the energy spectra in astrophysical sources tend to be a little steeper than $1/E^2$



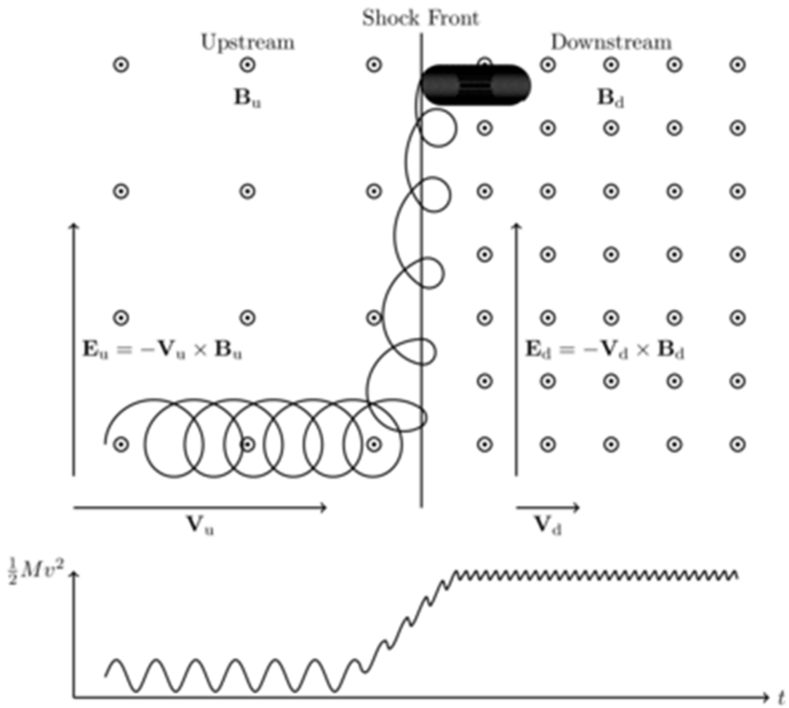
DSA: Conclusions

- Diffusive shock acceleration across non-relativistic shocks can accelerate particles to high energy with high efficiency
  - It produces a power-law spectrum ~E^{-2} with little dependence on details of shock
  - The maximum energy is of order *zeBVL* for a shock of speed *V* in magnetic field *B* in a source of size *L*
- Required magnetic field strengths and level of turbulence can be generated by non-linear effects in realistic simulations
  - There do appear to be unsolved issues in getting the energy spectrum exactly right, but these calculations require extremely sophisticated simulations with high CPU requirements, so future work may resolve this

ACCELERATION MECHANISMS

Shock Drift Acceleration (SDA)

SDA vs DSA



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- DSA takes place at quasi-parallel shocks
  - it does not happen at perpendicular shocks because the magnetic fields tend to prevent repeated shock crossings
- In quasi-perpendicular shocks, particles—especially electrons—can be trapped in the vicinity of the shock front
  - the particle is accelerated by the effective electric field in the shock front; the energy gain is proportional to the distance travelled along the front
- SDA usually only increases particle energy by ~factor 10
  - not an effective mechanism for CR acceleration, but may solve injection problem

ACCELERATION MECHANISMS

Acceleration at Relativistic Shocks

Relativistic shocks

- In many astrophysical sources, e.g. GRBs and AGN jets, there is good evidence for relativistic shocks ($V \sim c$)
  - acceleration in these shocks differs from DSA, because on return to the upstream rest frame there isn't time for the fast particles to become isotropic before the shock catches them
    - also, must consider beaming effects: cos θ will look very different to upstream and downstream observers
  - relativistic shock jump conditions (for a plane parallel shock)

$$\gamma_1 \rho_1 \beta_1 = \gamma_2 \rho_2 \beta_2;$$
$$\gamma_1^2 w_1 \beta_1^2 + p_1 = \gamma_2^2 w_2 \beta_2^2 + p_2;$$
$$\gamma_1^2 w_1 \beta_1 = \gamma_2^2 w_2 \beta_2$$

    - where enthalpy $w = \mathcal{E} + p$
  - equation of state $p = (\hat{\gamma} - 1)(\mathcal{E} - \rho c^2)$
    - $\hat{\gamma}$ is the effective ratio of specific heats

Highly relativistic case

- Take case where p_1 is negligible and $\mathcal{E}_2 \gg \rho_2 c^2$
  - then we have $p_2 \simeq (\hat{\gamma} - 1)\mathcal{E}_2$ and
    $$\gamma_1^2 w_1 \beta_1^2 = \gamma_2^2 w_2 \beta_2 \beta_1 \simeq \gamma_2^2 w_2 \beta_2^2 + (\hat{\gamma} - 1)\mathcal{E}_2$$
  - Substituting $w_2 = \mathcal{E}_2 + p_2 \simeq \hat{\gamma}\mathcal{E}_2$ and taking $\beta_1 \simeq 1$ (in the shock rest frame, the unshocked gas is moving at essentially *c*), we get
    $$\frac{\beta_2}{1 + \beta_2} \simeq \frac{\hat{\gamma} - 1}{\hat{\gamma}}$$
    and therefore $\beta_2 = \hat{\gamma} - 1$ (= ⅓ for a fully relativistic gas)
- Also, write $\beta_1 = 1 - \epsilon$ where $\epsilon \ll 1$
  - then as
    $$\beta_{\mathrm{rel}} = \frac{\beta_1 - \beta_2}{1 - \beta_1 \beta_2} \simeq 1 - 2\epsilon$$
  - we conclude that $\gamma_{\mathrm{rel}} \simeq 1/2\sqrt{\epsilon} = \gamma_1/\sqrt{2}$

Highly relativistic shocks

- Consider highly relativistic particle which crosses from upstream to downstream and back
  - in upstream rest frame its energy gain is given by
    $$\frac{E_f}{E_i} = \frac{1 - \beta_{\rm rel}\mu_1}{1 - \beta_{\rm rel}\mu_2} = \frac{1}{2}\gamma_s^2(1 - \beta_{\rm rel}\mu_1)(1 + \beta_{\rm rel}\bar{\mu}_2)$$
    - where $\mu_i \equiv \cos\theta_i$, and the bar indicates measurement in the downstream rest frame (other quantities measured in upstream rest frame)
    $$\mu_2 = \frac{\bar{\mu}_2 + \beta_{\rm rel}}{1 + \beta_{\rm rel}\bar{\mu}_2}$$
  - upstream particles will be overtaken by the shock if $-1 \le \beta_1\mu_1 < \beta_s$
    - if upstream particle is a particle of the upstream bulk gas, $\beta_1 \ll 1$ and $E_i \simeq mc^2$, giving $E_f = \frac{1}{2}\gamma_s^2(1 + \beta_{\rm rel}\bar{\mu}_2)mc^2$ and $\frac{2}{3}\gamma_s^2 < \frac{E_f}{E_i} \le \gamma_s^2$
    - if upstream particle was already relativistic, particle flux for given μ_1 is $\propto (\beta_s - \mu_1) \simeq (1 - \mu_1)$, giving $\frac{E_f}{E_i} = \frac{2}{3}\gamma_s^2(1 + \beta_{\rm rel}\bar{\mu}_2)$ and $\frac{8}{9}\gamma_s^2 < \frac{E_f}{E_i} \le \frac{4}{3}\gamma_s^2$

Highly relativistic shocks

- From previous, first return crossing leads to energy gain of factor $\sim\gamma_s^2$
  - this can be very large, as bulk γ factors for GRBs can be 10^2 or 10^3
- For subsequent shock crossing, must have $\bar{\mu}_2 > \frac{1}{3}$
  - this transforms to $\mu_2 > \beta_s = 1 - (1/\gamma_s)^2$, corresponding to $\sin\theta = \frac{1}{\gamma_s}$
  - to recross shock must scatter out of this cone, but relativistic scattering can only change angle by another $1/\gamma_s$, giving $\theta_2 < \frac{1}{\gamma_s} < \theta_1$
  - this gives a large escape probability, *P* ~ ⅓, and small energy gain

$$\frac{E_f}{E_i} \simeq \frac{1 + \frac{1}{2}\gamma_s^2\theta_1^2}{1 + \frac{1}{2}\gamma_s^2\theta_2^2} \simeq 2$$

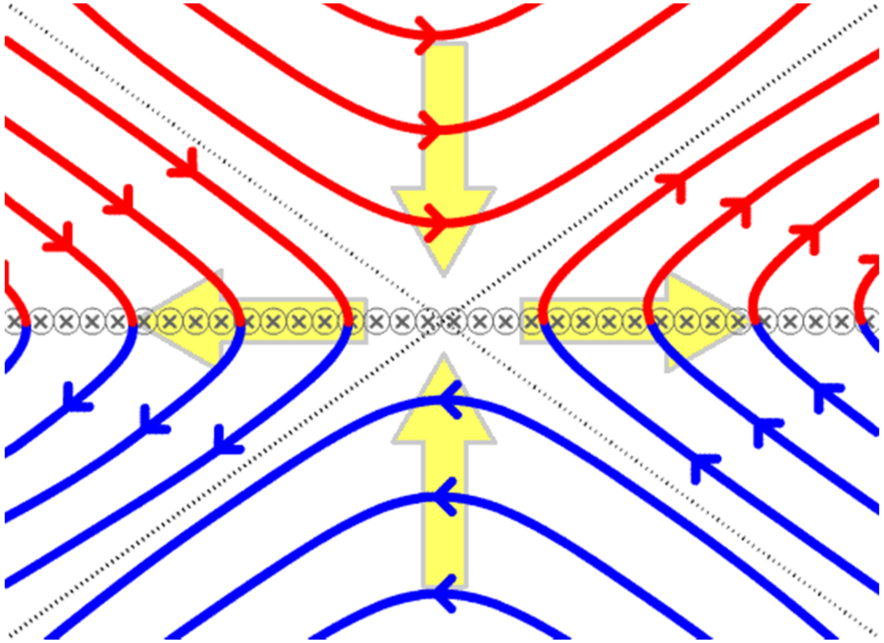
(assuming $\theta_2 \sim 2/\gamma_s$ and $\theta_1 \sim 1/\gamma_s$)

this turns out to imply a somewhat steeper power law, 2.3–2.4

ACCELERATION MECHANISMS

Magnetic Reconnection

Magnetic reconnection



- Magnetic reconnection occurs when magnetic field lines of opposed polarities are forced close together
    - resulting field configuration has lower energy, so energy is released in this process and can drive acceleration (and/or bulk flow of plasma)
- This is probably not appropriate for most astrophysical accelerators, but is implicated in solar flares and may be important near pulsars
    - in particular, it is *fast* and so can drive sudden γ-ray flares as observed, e.g., in Crab Nebula

ACCELERATION MECHANISMS

Propagation through the Galaxy

Propagation of CRs in Galaxy

- DSA predicts energy spectrum E^{-2}, maybe even less for realistic approach; relativistic shocks give $E^{-2.3}$ or so
  - observed spectrum is ~$E^{-2.7}$
  - is this a problem?
- Perhaps not: as CRs propagate through Galaxy, more energetic particles will have larger gyroradii and so are more likely to escape
  - can model this by a cylindrical "leaky box" with radius R ~ 15 kpc and height H ~ 3 kpc (height of Galactic magnetic field as estimated from synchrotron radiation)
  - escape time is then $\tau_{\rm esc} \simeq H^2/D(E)$ where $D(E)$ is diffusion coefficient for CRs of energy E
  - assume $D(E) = D_0 E^{-\delta}$, and take original spectrum $N_s(E) \propto E^{-\alpha}$

Propagation of CRs in Galaxy

- If rate of production by sources is $\mathcal{R}_s$, we have

$$N(E) \simeq \frac{N_s(E)\mathcal{R}_s}{2\pi R_d^2 H}\tau_{\text{esc}} \propto E^{-(\alpha+\delta)}$$

  for primary cosmic rays
- For secondary (spallation-produced) CRs, rate is

$$N_{\text{sec}}(E) \simeq N(E)\mathcal{R}_{\text{spall}}\tau_{\text{esc}} \propto E^{-(\alpha+2\delta)}$$

  - the extra factor of δ comes from the fact that spallation reactions occur throughout the lifetime of the CR in the galaxy
- Therefore comparison of secondary and primary cosmic rays should allow us to separate α and δ
  - results suggest $0.3 < \delta < 0.7$ and thus $2.0 < \alpha < 2.4$
  - models of CR anisotropy (lack of) prefer α at high end of range

Summary

You should read chapter 3 of the notes

You should know about

- *Fermi 2nd order mechanism*
- *shocks and shock jump conditions*
- *diffusive shock acceleration*
- *relativistic shocks*
- *magnetic reconnection*
- *propagation effects*

- Particle acceleration in astrophysical sources must involve magnetic fields
  - but scattering off magnetic fields in interstellar space is too inefficient
- Most popular mechanisms are diffusive shock acceleration and acceleration by relativistic shocks
  - both of these rely on favourable kinematics created by presence of shock
  - observational evidence does link shocks to acceleration (e.g. X-ray synchrotron emission from edges of supernova remnants)
- Magnetic reconnection may well be important in some sources
  - especially those associated with pulsars
- Propagation through Galaxy produces steeper energy spectrum

Next: some case studies

- solar system
- Galactic sources
- extragalactic sources

Notes chapter 4