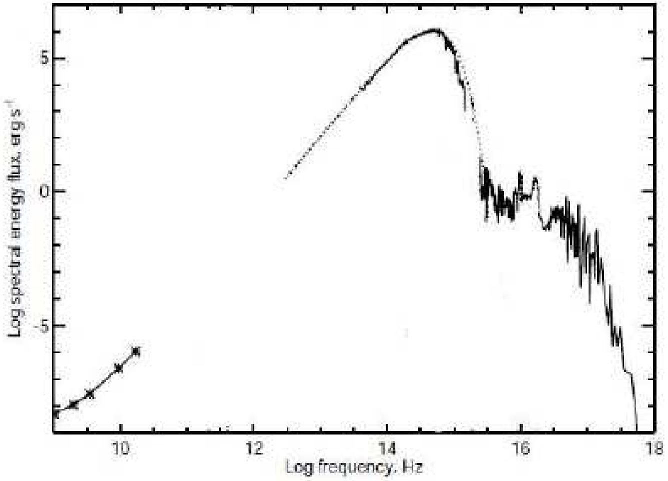
HIGH ENERGY PARTICLE ASTROPHYSICS

Radio Emission

Radio emission and particle astrophysics

Why are the lowest-energy photons relevant to high-energy particle astrophysics?

Because thermal radiation from stars is not significant in the radio waveband—bright radio emission is mostly ***non-thermal*** and diagnostic of high-energy particles.



http://www.cv.nrao.edu/course/astr534/Tour.html

RADIO EMISSION

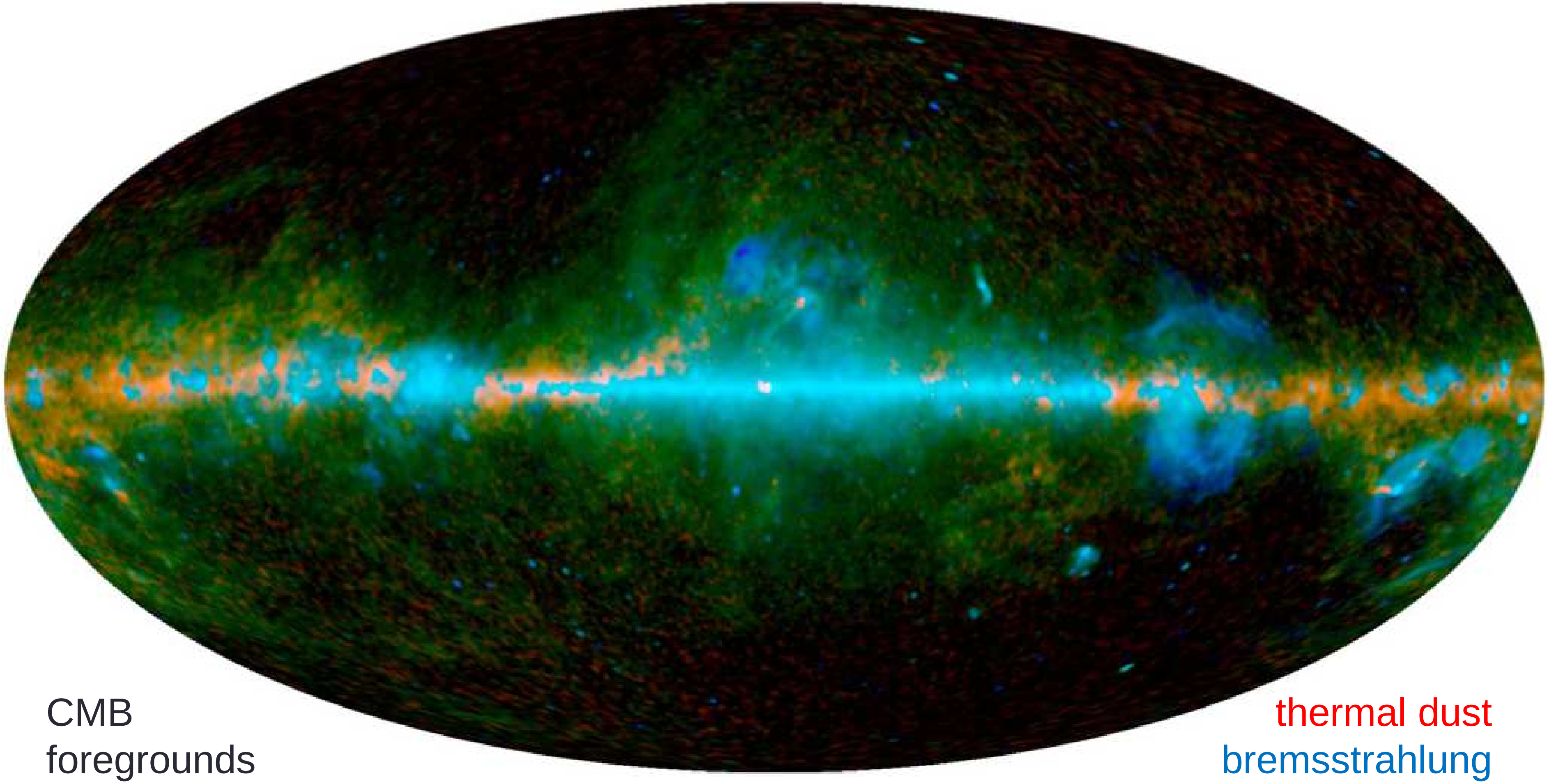
Emission Mechanisms

Radio emission mechanisms

- thermal emission from Galactic dust at 10-30 K
  - mostly far infra-red and submillimetre
- thermal emission from the CMB
  - submillimetre and microwave
- “spinning dust”
  - 5-30 mm, from very small, rapidly-spinning dust grains (important as foreground to CMB emission)
- line emission from gas
  - 21 cm (H I) plus many molecular lines
- **bremsstrahlung**
  - “braking radiation” from electron-ion interactions
- **synchrotron radiation**
  - from relativistic electrons in magnetic fields

these are of interest to us

Radio emission from Galaxy



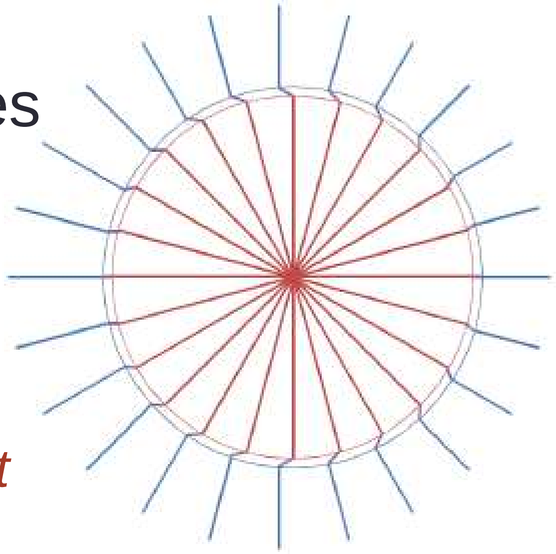
CMB
foregrounds
from 9-year WMAP analysis

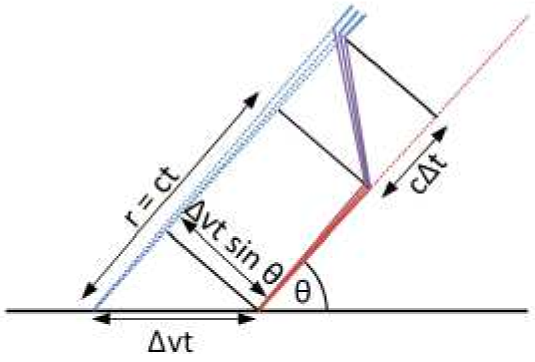
thermal dust
bremsstrahlung
synchrotron & spinning dust

RADIO EMISSION

Emission from an accelerated charge

Radiation from an accelerated charge





- If charge accelerates by Δ*v* in time Δ*t* (Δ*v* << *c*):
  - after time *t* there must be a “kink” in the field lines at *r* = *ct*
  - beyond this the field does not “know” about the acceleration
- Neglect aberration and assume field lines on either side of kink are radial
  - then the azimuthal field is given by $\frac{E_\theta}{E_r} = \frac{\Delta v t \sin\theta}{c\Delta t}$
  - so $E_\theta = \frac{Q \sin\theta}{4\pi\epsilon_0 c^2 r} \frac{\Delta v}{\Delta t}$

Power emitted

- Poynting vector $\boldsymbol{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$
  - for an electromagnetic wave in free space $E/B = c$ and **E** is perpendicular to **B**, so $S = E^2/c\mu_0 = c\varepsilon_0 E^2$
  - substitute for E from previous slide: then power through solid angle dΩ at angle θ is

$$P(\theta)d\Omega = \frac{Q^2|\ddot{\mathbf{r}}|^2 \sin^2\theta}{16\pi^2\epsilon_0 c^3 r^2}\, r^2 d\Omega$$

  - and we can integrate this over solid angle to get total power

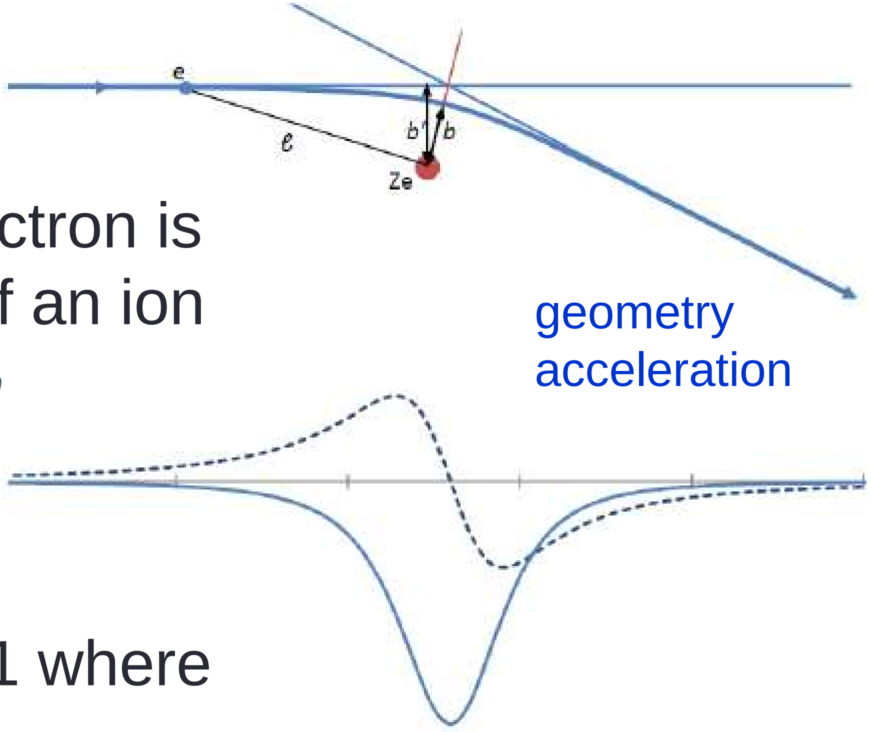
$$P_{\text{rad}} = \frac{Q^2|\ddot{\mathbf{r}}|^2}{6\pi\epsilon_0 c^3}$$

  - this is Lorentz invariant but $\ddot{\mathbf{r}}$ is measured in the instantaneous rest frame of the particle (**proper acceleration**)
  - in lab frame $|\ddot{\mathbf{r}}|^2 = \gamma^4\left(a_\perp^2 + \gamma^2 a_\parallel^2\right)$ ($\perp$ and $\parallel$ relative to **v**)

RADIO EMISSION

Bremsstrahlung

Bremsstrahlung



- Radiation emitted when an electron is deflected by the electric field of an ion
  - also known as ***free-free emission*** since the electron is not bound to the ion either before or after the scattering
- For ***radio frequencies*** $\omega\tau << 1$ where $\tau = 2b/\gamma v$
  - can neglect parallel acceleration since positive and negative cancel
  - can treat perpendicular acceleration as delta function with area $\Delta v_\perp$
- Fourier transform of a delta function is a constant
  - therefore Fourier transform of $a_\perp$ is $A_\perp(\omega) \approx \Delta v_\perp/(2\pi)^{1/2}$

Bremsstrahlung

- For a single electron

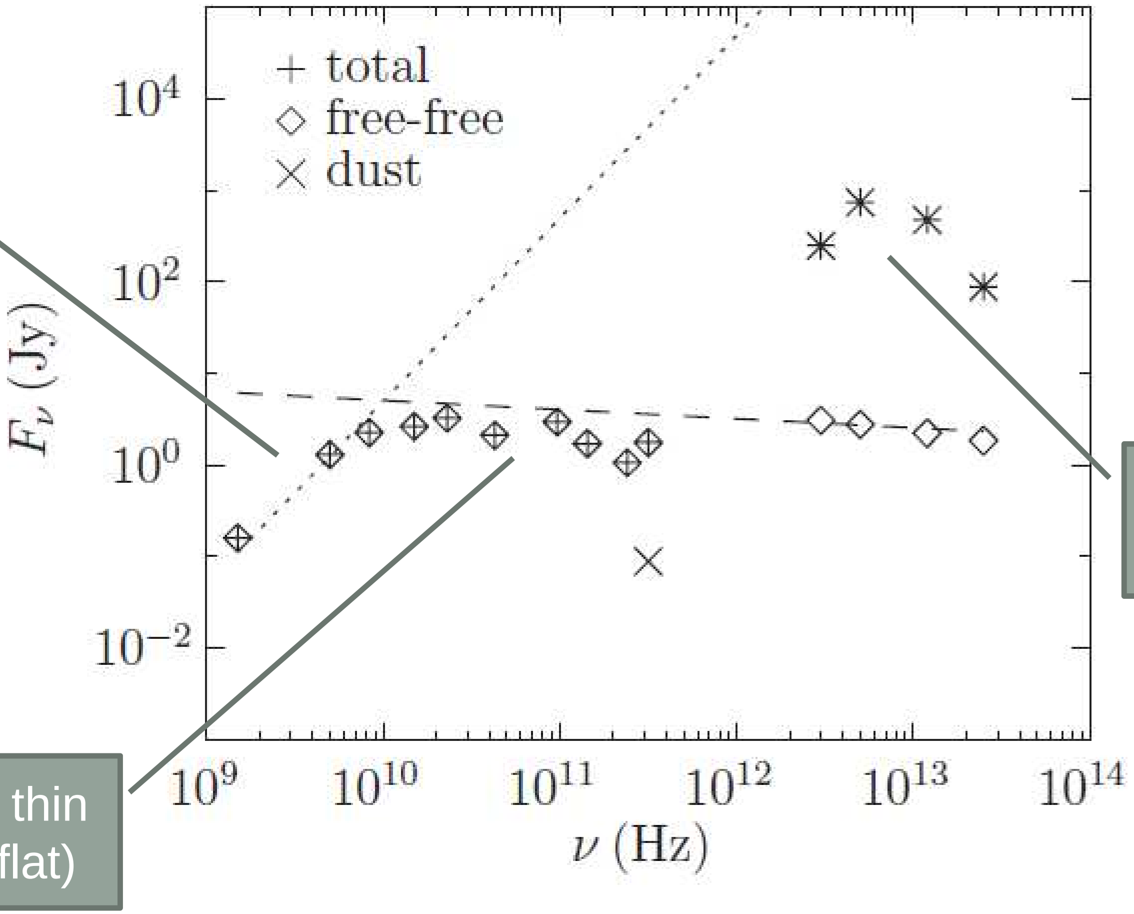
$$\Delta v_{\perp} = \frac{Ze^2}{4\pi\epsilon_0 m_e} \int_{-\infty}^{+\infty} \frac{\gamma b dt}{(b^2 + (\gamma v t)^2)^{3/2}} = \frac{2Ze^2}{4\pi\epsilon_0 m_e v b}$$

so

$$I(\omega) = \frac{e^2}{3\pi\epsilon_0 c^3} |A(\omega)|^2 = \frac{Z^2 e^6}{24\pi^4 \epsilon_0^3 c^3 m_e^2 v^2 b^2}$$

  - therefore spectrum of bremsstrahlung is flat at low frequencies, $\omega < \gamma v / b$ (at higher frequencies it falls off exponentially)
- Integrating this over a range of impact parameters *b* still gives a flat spectrum ∝ $\ln(b_{\max}/b_{\min})$ where $b_{\max}$ and $b_{\min}$ are inferred from the physics
- Integrating over a distribution of electron energies gives a flat spectrum for thermal, a power law for relativistic electrons

Typical bremsstrahlung spectrum



optically thick ($\propto \nu^2$, Rayleigh-Jeans)

optically thin (nearly flat)

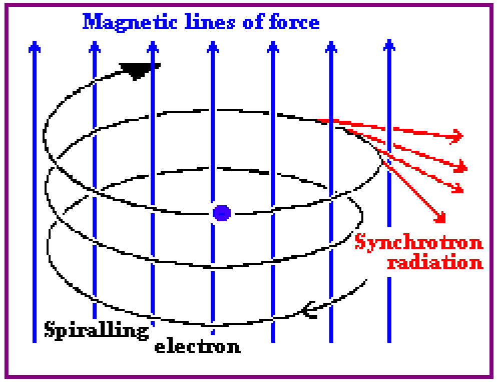
blackbody (dust)

Spectral energy distribution of a compact HII region

RADIO EMISSION

Synchrotron radiation

Synchrotron radiation



- Synchrotron radiation is emitted when a particle moves in a magnetic field

$$\frac{\mathrm{d}(\gamma m_0 \mathbf{v})}{\mathrm{d}t} = Ze(\mathbf{v} \times \mathbf{B})$$

$$\Rightarrow \gamma m_0 a_\perp = Zev_\perp B$$

  - particle moves in a spiral path with pitch angle given by tan $\theta = v_\perp / v_\parallel$ and radius

$$r_g = \frac{\gamma m_0 v \sin\theta}{ZeB}$$

  - total energy loss is

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{Z^4 e^4 B^2}{6\pi\epsilon_0 c} \frac{v^2}{c^2} \frac{\gamma^2}{m_0^2} \sin^2\theta$$

  - note that as $\gamma = E/m_0 c^2$ this is $\propto m_0^{-4}$: this is why we can neglect all particles other than electrons

Synchrotron radiation

- This can be written

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = 2c\sigma_{\mathrm{T}}U_{\mathrm{mag}}\beta^2\gamma^2\sin^2\theta$$

  - where the **Thomson cross-section**

$$\sigma_{\mathrm{T}} = \frac{e^4}{6\pi\epsilon_0^2c^4m_e^2}$$

  - and the energy density of the magnetic field $U_{\mathrm{mag}} = B^2/2\mu_0 = \frac{1}{2}\epsilon_0c^2B^2$

- Averaging over pitch angle (assumed isotropic) gives

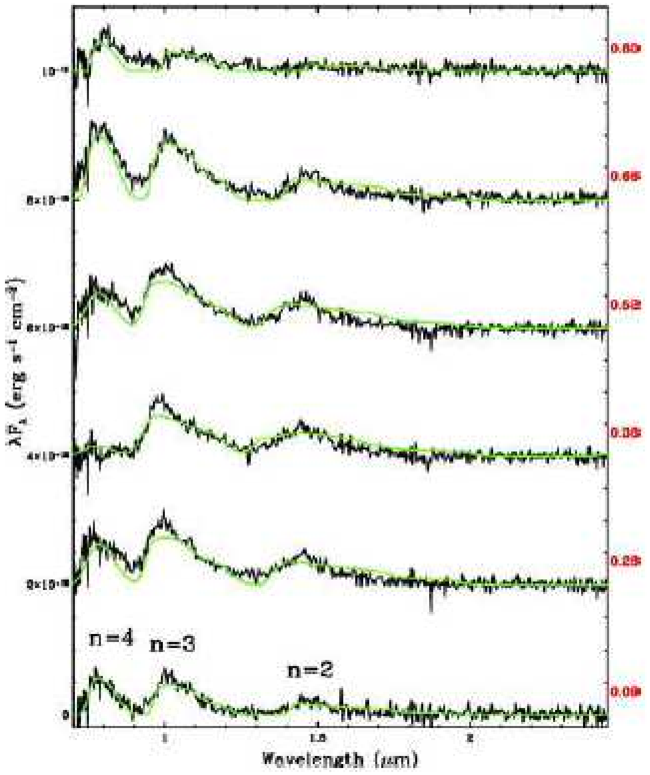
$$-\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{4}{3}c\sigma_{\mathrm{T}}U_{\mathrm{mag}}\beta^2\gamma^2$$

Cyclotron radiation

- Cyclotron radiation is emitted by non-relativistic or mildly relativistic electrons (γ ≈ 1)
  - at **cyclotron frequency** $\nu_g = eB/(2\pi m_e)$ for non-relativistic
  - at harmonics of gyrofrequency,

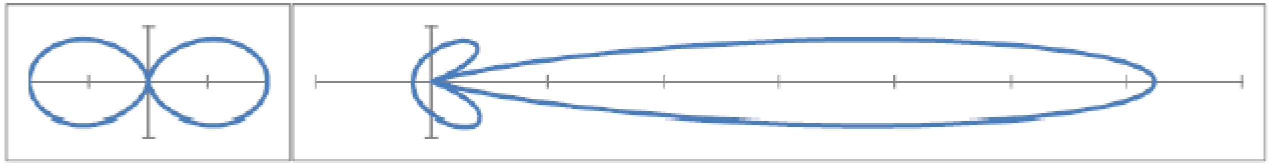
$$\nu_\ell = \frac{\ell}{1 - \beta_\parallel \cos\theta} \frac{eB}{2\pi\gamma m_e}$$

  for mildly relativistic
- Cyclotron radiation is polarised: linearly if **B** perpendicular to line of sight, circularly if **B** along line of sight, elliptically if in between
  - cyclotron lines are seen in some pulsars and close binary systems



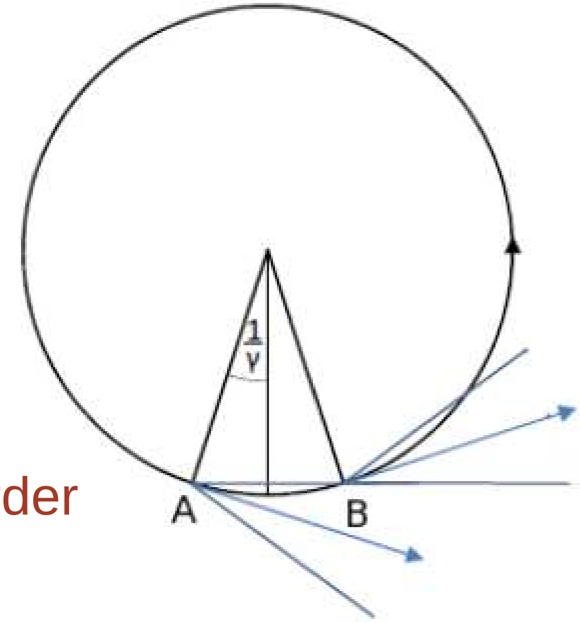
Synchrotron radiation and beaming

- Lorentz transformation of cos ϕ is

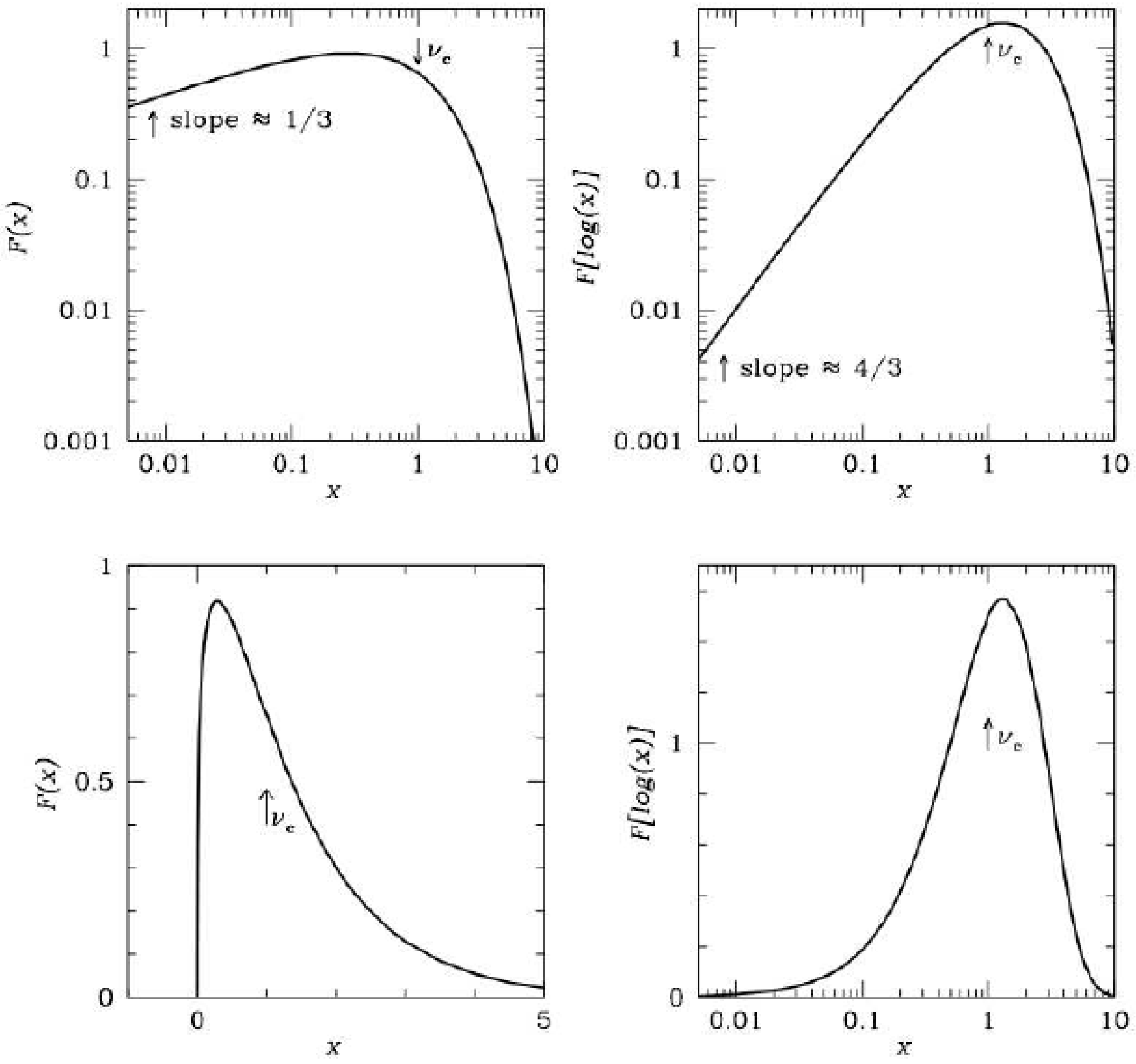


$$\cos\phi = \frac{\cos\phi' + \beta}{1 + \beta\cos\phi'}$$

  - for cos ϕ' = 0 this gives sin ϕ = 1/γ, i.e. radiation becomes concentrated in a narrow cone around particle direction of motion
  - radiation is only visible for time
    $$\Delta t = 1/\left(\gamma^2\omega_g \sin\theta\right)$$
    and hence has characteristic frequency of order
    $$\nu_s \simeq \gamma^2\omega_g \sin\theta$$
    where θ is the pitch angle



Synchrotron radiation: full spectrum



Estimate is correct order of magnitude and has correct dependence on γ

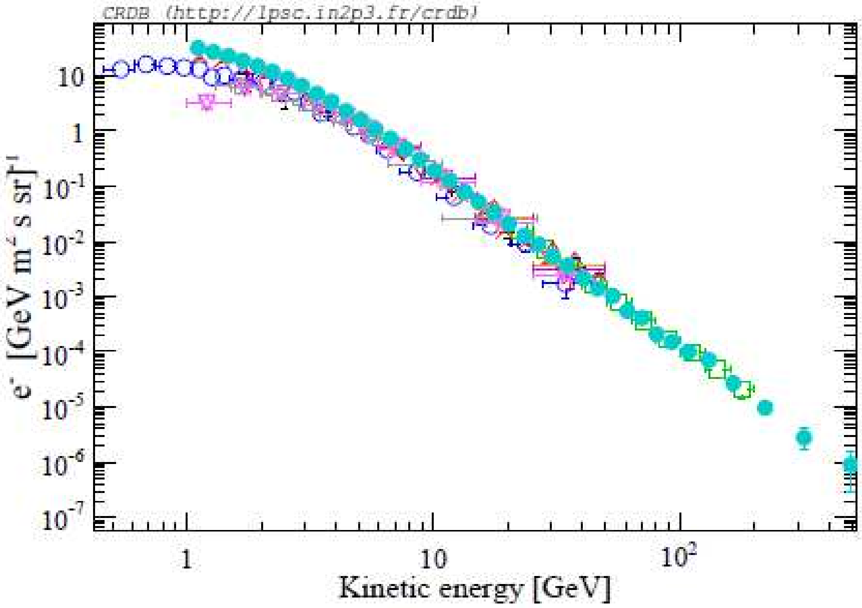
These spectra are in terms of

$$x = \frac{\nu}{\nu_c} = \frac{2\nu}{3\gamma^2 \nu_g \sin\theta}$$

Note that the spectrum is quite sharply peaked—often adequate to assume all radiation emitted at ν_c

This is for a single electron at fixed γ

Synchrotron radiation: power law



- Cosmic-ray electrons have power-law spectrum
- Assume all electrons radiate at frequency $\gamma^2 \nu_g$
- Spectral emissivity is

$$j_\nu \mathrm{d}\nu = -\frac{\mathrm{d}E}{\mathrm{d}t} N(E) \mathrm{d}E$$

  - $\mathrm{d}E/\mathrm{d}t \propto B^2\gamma^2$; $N(E) \propto E^{-\delta} \propto (\nu/\nu_g)^{-\delta/2}$; $\mathrm{d}E \propto \mathrm{d}\nu/(\nu\nu_g)^{1/2}$; $\nu_g \propto B$
- Keeping only dependence on ν and B, we have

$$j_\nu \propto B^{(\delta+1)/2} \nu^{-(\delta-1)/2}$$

  - if electron spectral index is ~3. expect synchrotron spectral index ~1
  - this is in reasonable agreement with observation
  - polarisation turns out to be $(\delta + 1)/\left(\delta + \frac{7}{3}\right)$: ~75% for $\delta \sim 3$

Synchrotron spectrum cut-offs

- Lifetime of electron of initial energy *E* is *E*/(−d*E*/d*t*)
  - this means that synchrotron spectrum will have a high-energy cut-off defined by the lifetime of the high-energy electrons
  - form of cut-off depends on how electrons are injected (over time vs instantaneously)
- Low-energy cut-off is introduced by source becoming opaque to its own radiation: **synchrotron self-absorption**
  - brightness temperature is defined as

$$T_b = \frac{\lambda^2}{2k}\frac{S_\nu}{\Omega}$$

flux

source solid angle

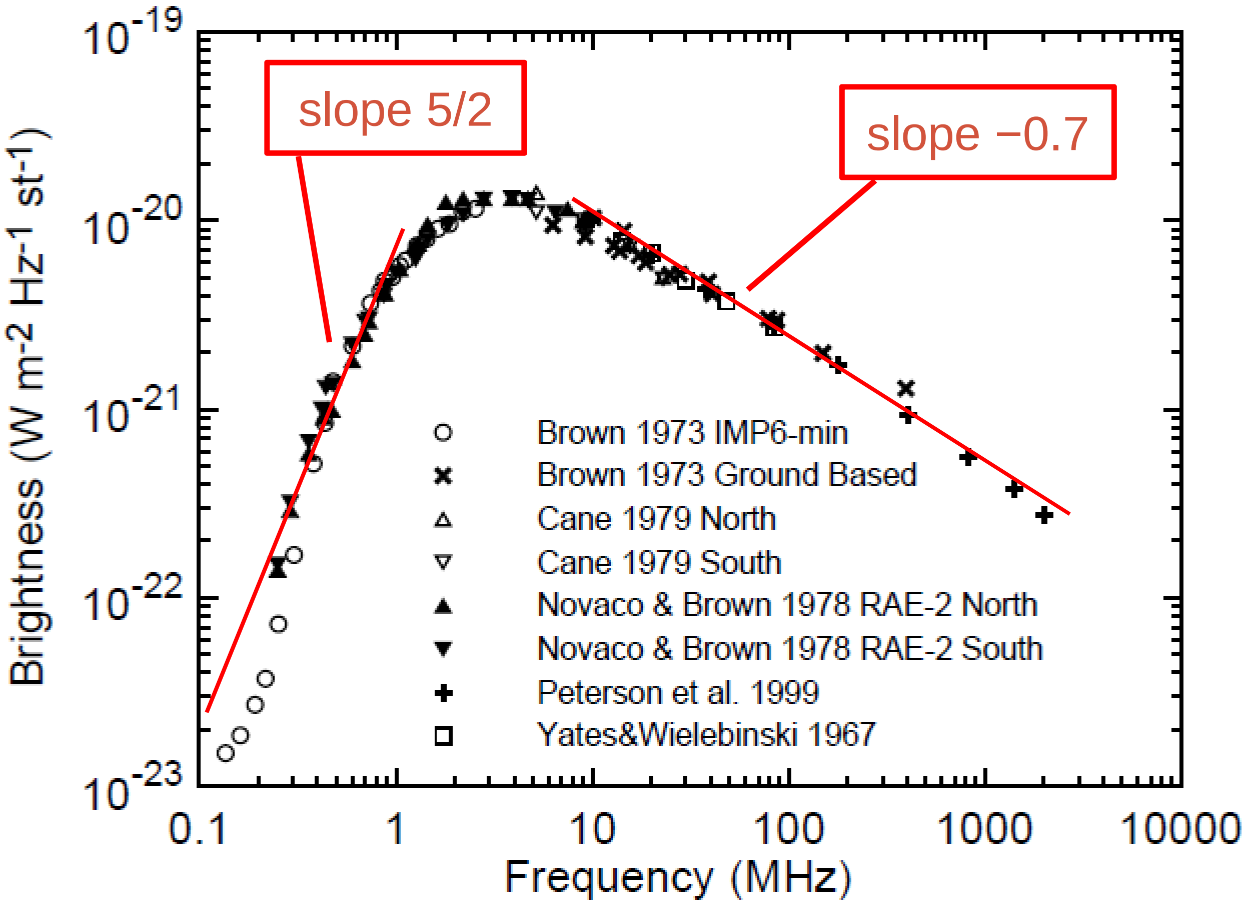
  - electron effective temperature is

$$T_e = \frac{\gamma m_e c^2}{3k} \simeq \frac{m_e c^2}{3k}\frac{\nu^{1/2}}{\nu_g^{1/2}}$$

  - equating these gives

$$S_\nu = 2 m_e \Omega \nu^{5/2} / \left(3\nu_g^{1/2}\right)$$

Synchrotron spectrum of Milky Way



Summary

You should read section 2.3 of the notes.

You should know about

- *radio emission mechanisms*
- *radiation from an accelerated charge*
- *bremsstrahlung*
- *synchrotron radiation*

- The atmosphere is transparent to radio emission (1 mm < λ < 10 m)
- There are many sources of radio emission, including thermal emission from dust and the CMB, line emission, and emission from accelerated charges
  - **bremsstrahlung** produces a flat spectrum with a ν^2 rise at low frequencies (self-absorption) and an exponential fall-off at high frequencies
  - **synchrotron radiation** produces a power law with spectral index ~1, with a $\nu^{5/2}$ rise at low frequencies and a cut-off at high frequencies from the electron energy
- Synchrotron radiation is diagnostic of the presence of relativistic electrons

Next: high-energy photon emission

- X-rays
- γ-rays

Notes section 2.4