PHY226 Tutorial Questions

Week 2: Binomial Series and Complex numbers

1. (a) Find the first few terms in the binomial series expansion of $(1-x)^{-1/2}$.

   (b) In the theory of special relativity, an object moving with velocity v has mass $m = \frac{m_o}{\sqrt{1-v^2/c^2}}$ where m_0 is the mass of the object at rest and c is the speed of light. The kinetic energy of the object is the difference between its total energy and its energy at rest: $K = mc^2 - m_0c^2$. Using the series found in (a), show that when $v \ll c$, the relativistic expression for K agrees with the classical expression $K = \frac{1}{2} mv^2$.

   (c) What is $z^{\frac{1}{3}}$ if $z = 64e^{\frac{i\pi}{3}}$?

Week 3: First and second order ODEs

2. Make sure you can state (hopefully instantly!) the general solution of the following 1st order ODEs:
   (a) $\frac{dx}{dt} = -\alpha x$, (b) $\frac{dx}{dt} = \beta x$, (c) $\frac{dx}{dt} = i\gamma x$, where α, β, γ are real positive constants.
   In what physical situations might equations of forms (a) and (b) occur?

3. What is the solution of the equation $\frac{dN(t)}{dt} = 0$ subject to the condition that $N = N_0$ when $t = 0$?

4. 2nd order ODEs. In lecture 4 we discussed the equations and solutions for the linear harmonic oscillator and unstable equilibrium.
   Now consider the 1D time independent Schrodinger equation for the spatial wavefunction *u(x)* of a particle of energy E in a constant potential V: $-\frac{\hbar^2}{2m}\frac{d^2u}{dx^2} + Vu = Eu$.
   Which form does this equation have, and what do its solutions look like, (i) for V < E? (ii) for V > E?

Week 4: ODEs cont.

5. (a) Express the complex function $Z(\omega) = -\omega^2 + 2i\gamma\omega + \omega_0^2$ in the form $Z(\omega) = |Z(\omega)|e^{i\phi}$ by finding expressions for $|Z(\omega)|$ and $\tan\phi$.

   (b) In the lecture we said that if *X(t)* is a solution of the equation $a\frac{d^2X}{dt^2} + b\frac{dX}{dt} + cX = Fe^{i\omega t}$ then $x(t) = \mathrm{Re}[X(t)]$ is the solution of $a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = F\cos\omega t$. Can you justify this?

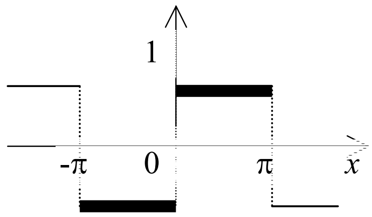
   (c) Using complex exponentials, find the steady state solution of the damped, driven oscillator equation $\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = F\cos\omega t$.

Week 5: Fourier Series

6. Show that the function shown has Fourier series

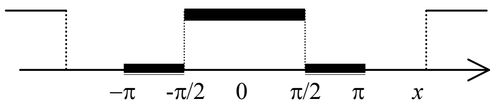
$$f(x) = \frac{4}{\pi}\sum_{n\,odd}\frac{1}{n}\sin nx$$

Explain why the series only contains sine terms.



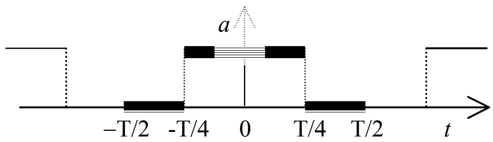
7. (a) Show that the function shown has Fourier series

$$f(x) = \frac{1}{2} + \frac{2}{\pi}\left(\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x - ...\right)$$



(b) Hence deduce that the function below has series

$$f(t) = \frac{a}{2} + \frac{2a}{\pi}\left(\cos\frac{2\pi t}{T} - \frac{1}{3}\cos\frac{6\pi t}{T} + \frac{1}{5}\cos\frac{10\pi t}{T} - ...\right)$$



8. A function is defined on the range [0, π]: $f(x) = x \qquad 0 < x < \pi$

(a) Sketch the *odd extension* of this function. Show that the corresponding half-range sine series is

$$f(x) = 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx$$

(b) Sketch the *even extension* of this function. Find the corresponding half-range cosine series.