

PHY226 – The math's module

Purpose of this module:

- Help you do well in the other modules
- Quantum mechanics, thermal physics, classical physics, optics, solid state physics...

PHY226 – Assessment

Three methods ...

- Homework 20%
- Problems classes 10%
- Exam 70%

PHY226 – Topics

Split in three

- Topics 1-4
- Topics 4-5
- Topics 6-9

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PHY226 – Topics 1-4

1. Revision of algebra
2. Complex numbers

3. Differential equations
4. Fourier series

Topics 5-8

Fourier transform, Convolution, Partial differential equations, Spherical co-ordinates

PHY226 – Weeks 1-4

Week 1

Monday 9 am
Monday 12:10
Thursday 12:10
Thursday 5 pm

Week 2

PHY202 instead

226 Problem class

Week 3

Monday 9 am
Thursday 5 pm

226 Homework set

Week 4

Monday 9 am
Thursday 5 pm

Lecture 1

Revision of Algebra

Purpose of lecture:

- **Remind you of what math's is**
- **Binomial and Taylor series**
- **Trig and Hyperbolic functions**

Revision Quiz...1

Simplify

$$\ln a - \ln b$$

Revision Quiz...1 (ans)

$$\ln a - \ln b = \ln(a / b)$$

Revision Quiz...2

Simplify

$$\ln a + \ln b$$

Revision Quiz...2 (ans)

$$\ln a + \ln b = \ln(ab)$$

Revision Quiz...3

Simplify

$$\ln(10^w)$$

Revision Quiz...3 (ans)

$$\ln(10^w) = w \ln 10$$

Revision Quiz...4

Simplify

$$e^a e^b / e^{-b}$$

Revision Quiz...4 (ans)

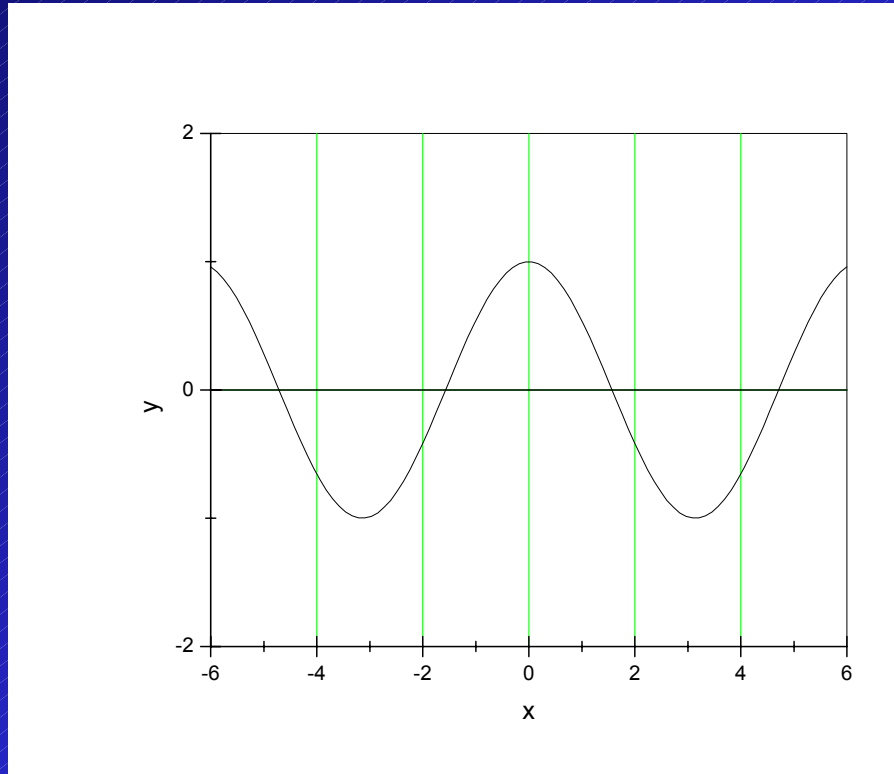
$$e^a e^b / e^{-b} = e^{(a+2b)}$$

Revision Quiz...5

Plot

$$\cos(-x)$$

Revision Quiz...5 (ans)



EVEN

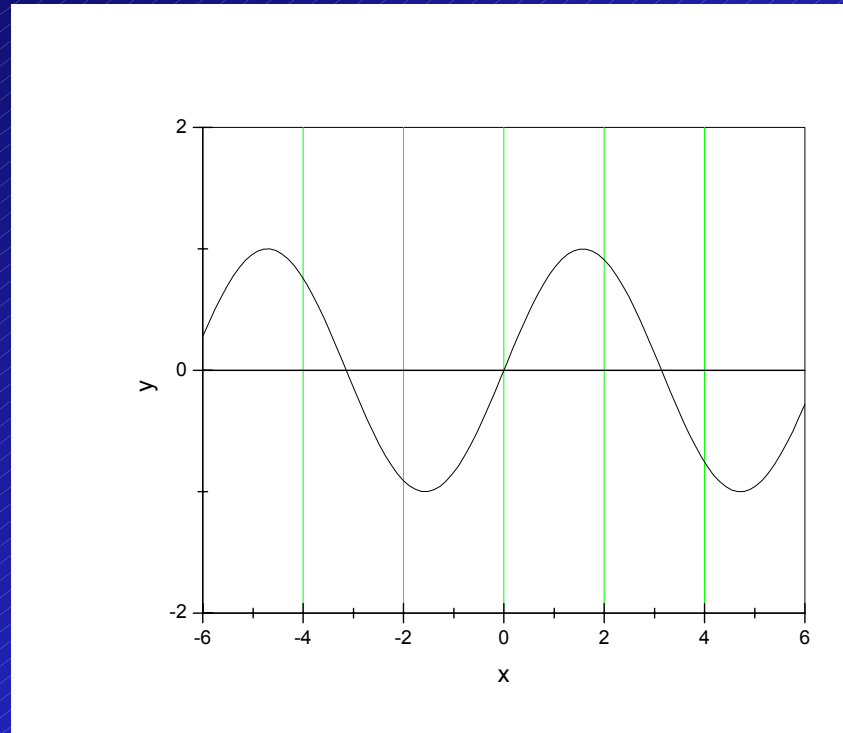
$$\cos(-x) = \cos(x)$$

Revision Quiz...6

Plot

$$\sin(-x)$$

Revision Quiz...6 (ans)



ODD

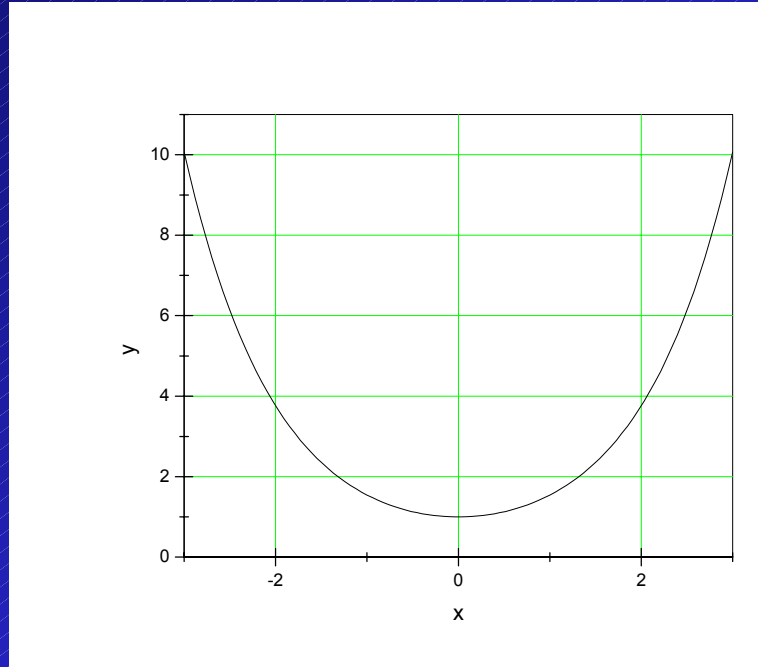
$$\sin(-x) = -\sin(x)$$

Revision Quiz...7

What is

$\cosh x$

Revision Quiz...7 (ans)



EVEN

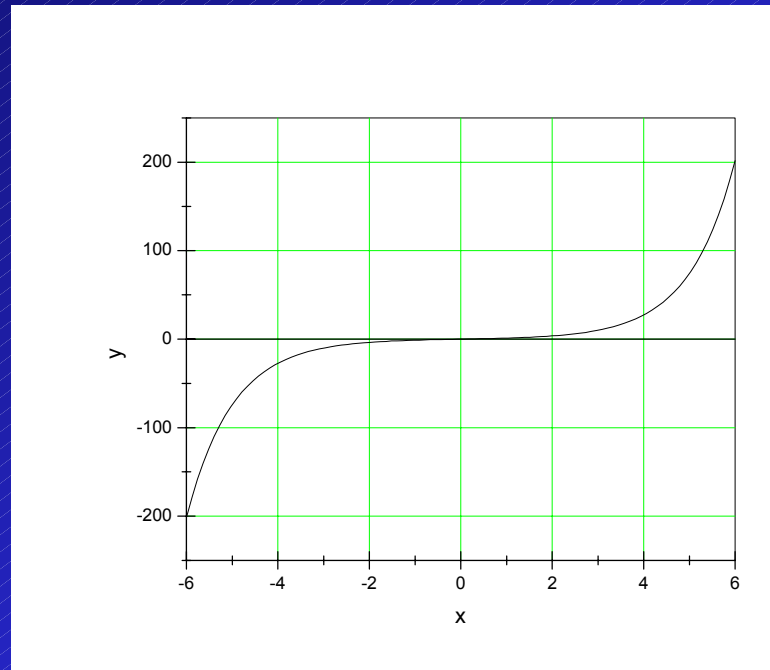
$$\cosh x = \frac{1}{2} \left(e^x + e^{-x} \right)$$

Revision Quiz...8

What is

$\sinh x$

Revision Quiz...8 (ans)



ODD

$$\sinh x = \frac{1}{2} \left(e^x - e^{-x} \right)$$

Hand out the notes

And introduce the website

<http://www.sheffield.ac.uk/physics/teaching/phy226/index.html>

PHY226 – Assessment

Three methods ...

- Homework 20%
- Problems classes 10%
- Exam 70%

Revision Quiz...9

Evaluate to three terms
using the binomial expansion

$$\frac{1}{1+x}$$

Revision Quiz...9

Binomial expansion:

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$$

Revision Quiz...9 (ans)

$$(1+x)^{-1} = 1 + (-1)x + (-1)(-2)\frac{x^2}{2} + (-1)(-2)(-3)\frac{x^3}{3.2} + ..$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + ..$$

...example 1.1 in the notes...

Revision Quiz...10

Evaluate to three terms
using a Taylor series

$$e^{ax}$$

Revision Quiz...10

Taylor series:

$$f(x+a) = f(a) + f'(a)x + \frac{f''(a)x^2}{2!} + \dots$$

Maclaurin series:

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \dots$$

Revision Quiz...10 (ans)

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \dots$$

$$\left. \begin{aligned} f(x) &= e^{\alpha x} \\ f'(x) &= \alpha e^{\alpha x} \\ f''(x) &= \alpha^2 e^{\alpha x} \end{aligned} \right\}$$

$$f(x) = e^{\alpha x} = e^0 + \alpha e^0 x + \frac{\alpha^2 e^0 x^2}{2!} + \frac{\alpha^3 e^0 x^3}{3!} + \dots$$

$$e^{\alpha x} = 1 + \alpha x + \frac{\alpha^2 x^2}{2} + \frac{\alpha^3 x^3}{6} + \dots$$

...example 1.3 in the notes...

Revision Quiz...11

Evaluate to two terms using
the binomial expansion

$$(\sin \theta + \cos \theta)^{15}$$

Revision Quiz...11 (ans)

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$$

$$(\sin\theta + \cos\theta)^{15} = (\cos\theta(\tan\theta + 1))^{15} = \cos^{15}\theta(1 + \tan\theta)^{15}$$

$$(1 + \tan\theta)^{15} = 1 + 15\tan\theta + \frac{(15)(14)}{2}\tan^2\theta + \dots$$

$$(1 + \tan\theta)^{15} = 1 + 15\tan\theta + 105\tan^2\theta + \dots$$

$$\cos^{15}\theta(1 + \tan\theta)^{15} = \cos^{15}\theta + 15\cos^{15}\theta\tan\theta + 105\cos^{15}\theta\tan^2\theta + \dots$$

...example 1.2 in the notes...

Revision Quiz...12

Evaluate to two terms
using a Taylor series

$$\tan\left(\frac{\pi}{4} + x\right)$$

Revision Quiz...12 (ans)

$$f(x+a) = f(a) + f'(a)x + \frac{f''(a)x^2}{2!} + \dots$$

$$\tan\left(\frac{\pi}{4} + x\right) = \tan\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)x = 1 + 2x$$

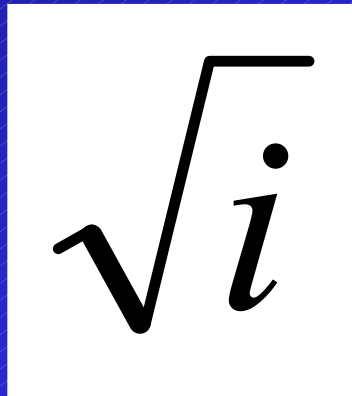
$$f\left(\frac{\pi}{4}\right) = 1$$

$$f'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right) = 2$$

...example 1.4 in the notes...

Revision Quiz...13

Evaluate



A white square containing the mathematical expression \sqrt{i} in black. The square root symbol is on the left, and the letter i is on the right, both in a serif font.

Revision Quiz...13 (ans)

$$\text{let } z = \sqrt{i} = a + ib$$

$$z^2 = i = (a + ib)^2 = a^2 - b^2 + 2abi$$

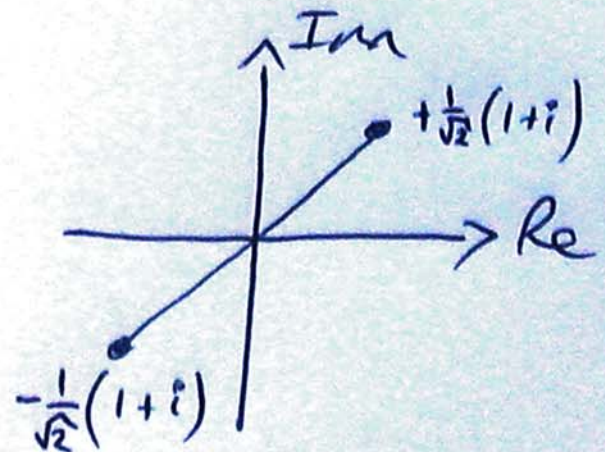
Equate real and imaginary parts.

$$a^2 - b^2 = 0 \Rightarrow a = b$$

$$2ab = 1 \Rightarrow 2a^2 = 1$$

$$a = b = \pm \frac{1}{\sqrt{2}}$$

$$z = \pm \frac{1}{\sqrt{2}}(1 + i)$$



Revision Quiz...14

True or false?

$$e^{i\pi} = -1$$

Revision Quiz...14

Euler's formula

$$e^{i\theta} = \cos \theta + i \sin \theta$$

De Moivre's theorem

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

Revision Quiz...14 (ans)

$$e^{i\pi} = \cos\pi + i\sin\pi = -1$$

need to prove $e^{ix} = \cos x + i\sin x$

Expand as Taylor series

$$e^{ix} = \dots$$

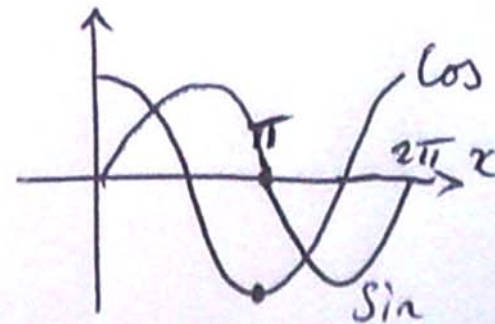
$$\cos x = \dots$$

$$i\sin x = \dots$$

} Sum and equate to e^x .

$$\cos\pi = -1$$

$$\sin\pi = 0$$



Prove...

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Lecture 1

Revision of Algebra

Remind you of

- Binomial and Taylor series
- Complex numbers
- Trig and Hyperbolic functions